

Solutions Manual

Second Edition

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Field and Wave Electromagnetics

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Chapter 3

Vector Analysis

- Ex. 1 a) $\vec{r}_0 = \frac{\vec{r}}{r} = \frac{\vec{r}_x + \vec{r}_y + \vec{r}_z}{\sqrt{r_x^2 + r_y^2 + r_z^2}} = \frac{1}{\sqrt{14}}(\vec{e}_1 + \vec{e}_2 + \vec{e}_3)$
 b) $|\vec{r} - \vec{r}_0| = |\vec{e}_1 + \vec{e}_2 + \vec{e}_3 - \vec{r}_0| = \sqrt{14 - 2\sqrt{14}} = \sqrt{10}$
 c) $\vec{r} \cdot \vec{r}_0 = 0 + 2(1 \cdot 1) + 0(1) = 2/\sqrt{14}$
 d) $\vec{e}_3 = \cos^2(\vec{r} - \vec{r}_0, \vec{e}_3) = \cos^2(\vec{e}_3 - \vec{r}_0, \vec{e}_3) = \cos^2 \alpha$
 e) $\vec{r} \cdot \vec{e}_3 = \vec{r} \cdot \frac{\vec{r}}{r} = \vec{r} \cdot \frac{1}{\sqrt{14}}(\vec{e}_1 + \vec{e}_2 + \vec{e}_3) = \frac{14}{\sqrt{14}}$
 f) $\vec{r} \times \vec{e}_3 = -\vec{e}_1 + \vec{e}_2 + \vec{e}_3 \times \vec{e}_3 = \vec{e}_2$
 g) $\vec{r} \cdot (\vec{r} \times \vec{e}_3) = (\vec{r} \times \vec{e}_3) \cdot \vec{r} = -\vec{e}_1 \cdot \vec{r}$
 h) $(\vec{r} \times \vec{e}_3) \times \vec{e}_3 = \vec{e}_3(\vec{e}_3 \cdot \vec{r}) - \vec{r}(\vec{e}_3 \cdot \vec{e}_3) = \vec{e}_3 r - \vec{e}_3 = \vec{e}_3(r - 1)$
 $\vec{r} \times (\vec{r} \times \vec{e}_3) = \vec{r}(\vec{e}_3 \cdot \vec{r}) - \vec{e}_3(\vec{r} \cdot \vec{r}) = \vec{e}_3 r - \vec{e}_3 r = \vec{0}$

Ex. 2 Let $\vec{C} = C_x \vec{e}_1 + C_y \vec{e}_2 + C_z \vec{e}_3$,

$$\text{where } C_x^2 + C_y^2 + C_z^2 = 1. \quad \textcircled{1}$$

$$\text{For } \vec{C} \perp \vec{A}: \vec{C} \cdot \vec{A} = 0 \implies C_x + 2C_y + 3C_z = 0. \quad \textcircled{2}$$

$$\text{For } \vec{C} \perp \vec{B}: \vec{C} \cdot \vec{B} = 0 \implies C_x + C_y + 2C_z = 0. \quad \textcircled{3}$$

Solving $\textcircled{1}$, $\textcircled{2}$, and $\textcircled{3}$ simultaneously, we obtain

$$C_x = \frac{1}{\sqrt{14}}, \quad C_y = \frac{1}{\sqrt{14}}, \quad C_z = \frac{1}{\sqrt{14}},$$

$$\text{and } \vec{C} = \frac{1}{\sqrt{14}}(\vec{e}_1 + \vec{e}_2 + \vec{e}_3).$$

Ex. 3 For $\vec{A}$ & $\vec{B}$ everywhere, $\vec{A} \times \vec{B} = \begin{vmatrix} \vec{e}_1 & \vec{e}_2 & \vec{e}_3 \\ x_1 & x_2 & x_3 \\ x_1 & x_2 & x_3 \end{vmatrix} = \vec{0}$,
 which requires that $\frac{x_1}{x_2} = \frac{x_2}{x_3} = \frac{x_3}{x_1}$.

Ex. 1.4 From $\vec{A} \cdot \vec{A} = \vec{A} \cdot \vec{C}$ we have $\vec{A} \cdot (\vec{B} - \vec{C}) = 0$. (1)

From $\vec{A} \cdot \vec{B} = \vec{A} \cdot \vec{C}$ we have $\vec{A} \cdot (\vec{B} - \vec{C}) = 0$. (2)

(1) implies $\vec{A} \perp (\vec{B} - \vec{C})$, and (2) implies $\vec{A} \perp (\vec{B} - \vec{C})$.

Since $\vec{A}$ is not a null vector, (1) and (2) cannot hold at the same time unless $(\vec{B} - \vec{C})$ is a null vector. Thus, $\vec{B} - \vec{C} = 0$, or $\vec{B} = \vec{C}$.

Ex. 1.5 Expand $\vec{A} \cdot (\vec{A} \times \vec{B}) = \vec{A} \cdot (\vec{B} \times \vec{C}) = \vec{B} \cdot (\vec{C} \times \vec{A})$.

or $\vec{A} \times \vec{B} = p\vec{A} + q\vec{B}$.

$$\therefore \vec{C} = \frac{1}{p} (p\vec{A} + q\vec{B}).$$

Ex. 1.6 Position vectors of the three corners:

$$\vec{OQ_1} = x_1\hat{i} + y_1\hat{j}, \quad \vec{OQ_2} = x_2\hat{i} + y_2\hat{j}, \quad \vec{OQ_3} = x_3\hat{i} + y_3\hat{j} + z_3\hat{k}$$

Vectors representing the three sides of the triangle:

$$\vec{Q_1Q_2} = \vec{OQ_2} - \vec{OQ_1} = x_2\hat{i} - x_1\hat{i}, \quad \vec{Q_2Q_3} = \vec{OQ_3} - \vec{OQ_2} = x_3\hat{i} - x_2\hat{i}, \quad \vec{Q_3Q_1} = \vec{OQ_1} - \vec{OQ_3} = x_1\hat{i} - x_3\hat{i}.$$

$$\therefore \vec{Q_1Q_2} \cdot \vec{Q_2Q_3} = 0. \quad \therefore \triangle Q_1Q_2Q_3 \text{ is a right triangle.}$$

$$\therefore \text{Area of triangle} = \frac{1}{2} |\vec{Q_1Q_2} \times \vec{Q_2Q_3}| = 17.1$$

Ex. 1.7



$$\vec{A_1} = \vec{B} + \vec{D}, \quad \vec{A_2} = \vec{B} - \vec{D}$$

$$\vec{A_1} \cdot \vec{A_2} = (\vec{B} + \vec{D}) \cdot (\vec{B} - \vec{D})$$

$$= \vec{B} \cdot \vec{B} - \vec{D} \cdot \vec{D} = 0$$

for a rhombus.

$$\therefore \vec{A_1} \perp \vec{A_2}$$

Ex. 11



Let A , B , and C denote the vertices of a triangle, and D and E be the midpoints of sides AB and AC , respectively. The following vector relations hold:

$$\vec{AD} = \frac{1}{2} \vec{AB}, \quad \vec{AE} = \frac{1}{2} \vec{AC}.$$

$$\begin{aligned} \vec{DE} &= \vec{AE} - \vec{AD} = \frac{1}{2} (\vec{AC} - \vec{AB}) \\ &= \frac{1}{2} \vec{BC}. \end{aligned} \quad \text{Q.E.D.}$$

Ex. 12 $\vec{a}_x = \vec{a}_x \cos \alpha + \vec{a}_y \sin \alpha,$

$$\vec{a}_y = \vec{a}_x \sin \beta + \vec{a}_y \cos \beta.$$

a) $\vec{a}_x = \vec{a}_y = \cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta.$

b) $\vec{a}_y = \vec{a}_x = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ \cos \beta & \sin \beta & 0 \\ \cos \alpha & \sin \alpha & 0 \end{vmatrix} = \vec{a}_x (\sin \alpha \cos \beta - \cos \alpha \sin \beta)$
 $= \vec{a}_x \sin(\alpha - \beta).$

$$\therefore \sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta.$$

Ex. 13



$$\vec{a} + \vec{b} + \vec{c} = 0.$$

$$\vec{a} \cdot \vec{a} : \vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c}.$$

$$\vec{c} \cdot \vec{a} : \vec{c} \cdot \vec{b} = \vec{b} \cdot \vec{c}.$$

$$\vec{b} \cdot \vec{a} : \vec{b} \cdot \vec{c} = \vec{a} \cdot \vec{c}.$$

Magnitudes relations:

$$AB \sin \alpha_2 = AC \sin \alpha_1 = BC \sin \alpha_3.$$

Then:

$$\frac{1}{\sin \alpha_1} = \frac{1}{\sin \alpha_2} = \frac{1}{\sin \alpha_3} \quad \left(\frac{1}{\sin \alpha_1} \right)$$

E.2.1)

$$P = -P, \quad r = r + d.$$

$$(E - P) \cdot (E - P) = (E + 0)(E + P) = 0.$$

$$\therefore (E - P) \perp (E - P).$$

E.2.2 Consider the line $L_1: \quad k_1 x + k_2 y = d_1$, which has a slope equal to $-k_1/k_2$. Derive the divided line passing through the origin and parallel to L_1 as $L_1': \quad k_1 x + k_2 y = 0$.

The position vector of a point (x, y) on L_1' is

$$\vec{r} = \vec{r}_x x + \vec{r}_y y.$$

If we introduce the vector $\vec{r} = \vec{r}_x k_1 + \vec{r}_y k_2$, we can write the equation of L_1' as

$$\vec{r} \cdot \vec{r} = 0.$$

Thus the vector $\vec{r}$ is $\perp$ to L_1 , and is normal to both L_1 and L_1' . It follows that the two lines L_1 and L_1' are perpendicular to each other if and only if their normal vectors $\vec{r}$ and $\vec{r}' = k_1 \vec{r}_x' + k_2 \vec{r}_y'$ are orthogonal. This which implies

$$k_1 k_1' + k_2 k_2' = 0, \quad \text{or} \quad \frac{k_1'}{k_2'} = -\frac{k_1}{k_2}.$$

That is, the slopes of lines L_1 and L_1' are the negative reciprocals of each other.

E.2.3 a) Letting the position vector of a point in the plane be

$$\vec{r} = \vec{r}_x x + \vec{r}_y y + \vec{r}_z z$$

and introducing the vector $\vec{r} = \vec{r}_x k_1 + \vec{r}_y k_2 + \vec{r}_z k_3$, we can write the plane equation as

$$\vec{r} \cdot \vec{r} = a \quad (a \text{ constant}).$$

This shows that the projection of the position vector to any point in the plane on $\vec{r}$ is a constant, and that $\vec{r}$ is a normal vector.

$$b) \quad \vec{r}_n = \frac{\vec{r}}{|\vec{r}|} = \frac{\vec{r}_x \hat{e}_x + \vec{r}_y \hat{e}_y + \vec{r}_z \hat{e}_z}{\sqrt{r_x^2 + r_y^2 + r_z^2}}.$$

c) The perpendicular distance from the origin to the plane is

$$\vec{r}_n \cdot \vec{r} = \frac{a}{|\vec{r}|}.$$

For our case, $a = R$, $|\vec{r}| = \sqrt{R^2 + R^2 + R^2} = R\sqrt{3}$,
and $\vec{r}_n \cdot \vec{r} = R/\sqrt{3}$.

P. 3.15 $\vec{A}_1 = -\vec{r}_y \hat{e}_x + \vec{r}_x \hat{e}_y, \quad \vec{A}_2 = -\vec{r}_x \hat{e}_x + \vec{r}_y \hat{e}_y,$
 $\vec{A}_3 = \vec{r}_x (\cos \phi) + \vec{r}_y (\sin \phi) + \vec{r}_z (\frac{1}{2}) = \vec{r}_x \frac{1}{2} + \vec{r}_y \frac{1}{2} + \vec{r}_z,$
 $\vec{A}_4 = \vec{A}_1 + \vec{A}_2 = \vec{r}_x \frac{1}{2} + \vec{r}_y \frac{1}{2} + \vec{r}_z, \quad |\vec{A}_4| = \sqrt{2},$
 $\vec{A}_1 \cdot \vec{A}_4 = \vec{A}_1 \cdot \frac{\vec{A}_4}{|\vec{A}_4|} = \frac{\vec{A}_1 \cdot \vec{A}_4}{\sqrt{2}} = \frac{1}{2} \sqrt{2}.$

P. 3.16 a) $x = r \cos \phi = 4 \cos(\frac{1}{2}\pi/3) = 2,$
 $y = r \sin \phi = 4 \sin(\frac{1}{2}\pi/3) = 2\sqrt{3},$
 $z = 3.$

b) $\vec{r} = (x^2 + y^2)^{1/2} = (4^2 + 3^2)^{1/2} = 5,$
 $\phi = \tan^{-1}(y/x) = \tan^{-1}(\sqrt{3}) = 60^\circ,$
 $\phi = \tan^{-1}(3/4).$

P. 3.17 a) $\vec{r}_1 = \vec{r}_0 \frac{r_0 \hat{e}_r + \frac{1}{2} \hat{e}_z}{\sqrt{r_0^2 + \frac{1}{4}}} = \vec{r}_0 \frac{\hat{e}_r}{2},$
 $\vec{A}_1 = \frac{1}{2} \left(\frac{r_0 \hat{e}_r}{\sqrt{r_0^2 + \frac{1}{4}}} \right) = r_0 \hat{e}_r.$

b) $\vec{r}_2 = r_0 \frac{1}{\sqrt{r_0^2 + \frac{1}{4}}} (\hat{e}_r + \frac{1}{2} \hat{e}_z) = \vec{r}_0 \hat{e}_r, \quad \vec{r}_3 = \frac{1}{2} \hat{e}_z + \frac{1}{2} \vec{r}_0 \hat{e}_r + \vec{r}_0 \hat{e}_z$
 $(\hat{e}_r \cos \phi \hat{e}_x + \hat{e}_r \sin \phi \hat{e}_y) = \cos \phi \hat{e}_x + \frac{1}{2} \frac{\hat{e}_z}{\sqrt{r_0^2 + \frac{1}{4}}} + \frac{1}{2} \hat{e}_z.$

$$\begin{aligned}
 \underline{2.2-17} \quad \vec{F}_x &= \vec{F}_x \sin \phi \cos \phi + \vec{F}_y \cos \phi \sin \phi + \vec{F}_z \cos \phi = \frac{\vec{F}_x \sin \phi + \vec{F}_y \cos \phi + \vec{F}_z}{\sqrt{\sin^2 \phi + \cos^2 \phi + 1}} \\
 \vec{F}_y &= \vec{F}_y \sin \phi \cos \phi + \vec{F}_z \cos \phi \sin \phi - \vec{F}_x \sin \phi = \frac{\vec{F}_y \sin \phi + \vec{F}_z \cos \phi - \vec{F}_x}{\sqrt{\sin^2 \phi + \cos^2 \phi + 1}} \\
 \vec{F}_z &= \vec{F}_x \sin \phi + \vec{F}_y \cos \phi = \frac{\vec{F}_x \sin \phi + \vec{F}_y \cos \phi}{\sqrt{\sin^2 \phi + \cos^2 \phi}}
 \end{aligned}$$

2.2-18

$$\begin{aligned}
 \text{a) } \vec{F}_x \cdot \vec{F}_y &= \sin \phi, & \text{b) } \vec{F}_x \cdot \vec{F}_y &= \cos \phi \sin \phi, & \text{c) } \vec{F}_x \cdot \vec{F}_z &= \vec{F}_x \sin \phi \\
 \text{d) } \vec{F}_y \cdot \vec{F}_z &= \sin \phi, & \text{e) } \vec{F}_x \cdot \vec{F}_z &= \cos \phi \sin \phi, & \text{f) } \vec{F}_y \cdot \vec{F}_z &= \cos \phi \\
 \text{g) } \vec{F}_x \cdot \vec{F}_z &= \vec{F}_y \sin \phi, & \text{h) } \vec{F}_x \cdot \vec{F}_y &= -\sin \phi, & \text{i) } \vec{F}_x \cdot \vec{F}_z &= \vec{F}_y \cos \phi
 \end{aligned}$$

$$\underline{2.2-19} \quad \begin{aligned} \vec{F} \cdot d\vec{r} &= [\vec{F}_x \cos \phi + \vec{F}_y (2x - y^2) - \vec{F}_z \cos \phi] \cdot [dx \cos \phi + dy \sin \phi] \\ &= \cos \phi \, dx + (2x - y^2) \, dy \end{aligned}$$

a) Along direct path D. The equation of D is $y = \frac{1}{2}(x - 1)$.

$$\begin{aligned}
 \int_{\text{over D}} \vec{F} \cdot d\vec{r} &= \int_0^1 \left[\cos \phi \, dx + (2x - y^2) \, dy \right] \\
 &= \int_0^1 \frac{1}{4} x \cos \phi \, dx + \int_0^1 (2x + 1 - x^2) \, dy \\
 &= -2\phi + 4\phi = 2\phi
 \end{aligned}$$

b) Along path D. This path lies first along the line $x = 1$, from $\phi = 0$ to $\phi = \pi/2$, then to $\vec{F} \cdot d\vec{r} = y^2 dy$. From $\phi = \pi/2$ to $\phi = \pi$, $dy = 0$, $\vec{F} \cdot d\vec{r} = dx$. Hence,

$$\begin{aligned}
 \int_{\text{over D}} \vec{F} \cdot d\vec{r} &= \int_0^{\pi/2} (2x - y^2) \, dy + \int_{\pi/2}^{\pi} 1 \, dx = 2\phi + \pi = \pi \\
 &\neq \int_{\text{over D}} \vec{F} \cdot d\vec{r} \text{ using Stokes' Theorem}
 \end{aligned}$$

$$\underline{\text{Ex 2-10}} \quad \int_C \vec{F} \cdot d\vec{r} = \int_0^{\pi/2} (y \, dx + x \, dy)$$

$$\text{a) } x = 2y^2, \quad dx = 4y \, dy, \quad \int_0^{\pi/2} \vec{F} \cdot d\vec{r} = \int_0^{\pi/2} (y^2 \, dy + 2y^2 \, dy) = 10$$

$$b) \quad \mathbf{r} = x\mathbf{i} + y\mathbf{j}, \quad d\mathbf{r} = dx\mathbf{i} + dy\mathbf{j}, \quad \int_C \mathbf{r} \cdot d\mathbf{r} = \int_0^1 \int_0^{2\pi} (x dx + y dy) r dr = 0.$$

Equal line integrals along two opposite paths do not necessarily imply a conservative field. $\mathbf{F}$ is a conservative field in this case because $\mathbf{F} = \nabla(x^2 + y^2)$.

Ex. 22

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} \cos t & \sin t \\ -\sin t & \cos t \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \cos t & \sin t \\ \sin t & \cos t \end{bmatrix} \begin{bmatrix} \cos t \\ \sin t \end{bmatrix}.$$

$$\dot{\mathbf{r}} = x_1 \cos t \mathbf{i} + x_2 \sin t \mathbf{j},$$

$$\mathbf{r} \cdot d\mathbf{r} = x_1 dx + x_2 dy = r^2 \cos t dt.$$

$$\oint_C \mathbf{r} \cdot d\mathbf{r} = \int_0^{2\pi} r^2 \cos t dt = 0, \quad \oint_C \mathbf{r} \cdot d\mathbf{r} = \int_0^{2\pi} r^2 \sin t dt = 0.$$

There is no change in r for t from t_1 to t_2 .

$$\therefore \int_C \mathbf{r} \cdot d\mathbf{r} = r^2 \int_{t_1}^{t_2} \cos t dt = 0.$$

$$\begin{aligned} \text{Ex. 23. a) } \oint_C \mathbf{r} \cdot d\mathbf{r} &= \left(x_1 \left(\frac{1}{2} \cos \frac{\pi}{2} \right) + x_2 \left(\frac{1}{2} \sin \frac{\pi}{2} \right) \right) \left(\frac{1}{2} \cos \frac{\pi}{2} \right) \\ &\quad + \left(x_1 \left(\frac{1}{2} \sin \frac{\pi}{2} \right) + x_2 \left(\frac{1}{2} \cos \frac{\pi}{2} \right) \right) \left(\frac{1}{2} \sin \frac{\pi}{2} \right) \\ &= 0. \end{aligned}$$

$$\oint_C \mathbf{r} \cdot d\mathbf{r} = - \left(x_1 \frac{1}{2} + x_2 \frac{1}{2} \right) e^t = - \left(x_1 \cos t + x_2 \sin t \right).$$

$$b) \quad \mathbf{r} \cdot d\mathbf{r} = (x_1 + x_2) dx, \quad d\mathbf{r} = \frac{1}{\sqrt{2}} (x_1 + x_2) dt.$$

$$\therefore \oint_C \mathbf{r} \cdot d\mathbf{r} = \frac{1}{\sqrt{2}} \left(\frac{1}{2} + \frac{1}{2} \right) e^t = 0.5 e^t.$$

Ex. 24 On the surface of the sphere, $\mathbf{r} = R$.

$$\begin{aligned} \oint_C \mathbf{r} \cdot d\mathbf{r} &= \int_0^{2\pi} \int_0^{\pi} (R) \sin \theta (R^2 \sin \theta) d\theta d\phi \\ &= \int_0^{2\pi} \int_0^{\pi} R^3 \sin^2 \theta d\theta d\phi = 2\pi R^3. \end{aligned}$$

Ex. 25 The first step is to find the equation for the unit normal $\mathbf{u}_n = \mathbf{r}_1 \mathbf{i} + \mathbf{r}_2 \mathbf{j} + \mathbf{r}_3 \mathbf{k}$ to the plane surface. This gives four values points of the surface and the following four equations



Corner $(0,0,0)$
 $\vec{r} = a\vec{e}_1 + b\vec{e}_2 + c\vec{e}_3$ (1)

Corner $(a,0,0)$
 $\vec{r} = (a-x)\vec{e}_1 + b\vec{e}_2 + c\vec{e}_3$ (2)

Corner $(0,b,0)$
 $\vec{r} = a\vec{e}_1 + (b-y)\vec{e}_2 + c\vec{e}_3$ (3)

Corner $(0,0,c)$
 $\vec{r} = a\vec{e}_1 + b\vec{e}_2 + (c-z)\vec{e}_3$ (4)

The direction cosines satisfy the condition:
 $\vec{r} \cdot \vec{r} = a^2 + b^2 + c^2 = 1$ (5)

From (1)-(4) we obtain $\vec{r} \cdot \vec{r} = 1$, and $a-x = b = c = 0$.

Thus, $\vec{r} = \frac{1}{\sqrt{a^2+b^2+c^2}}(a\vec{e}_1 + b\vec{e}_2 + c\vec{e}_3)$ (in vector form)
 and $\int_V \vec{r} \cdot d\vec{V} = \frac{1}{\sqrt{a^2+b^2+c^2}} \int_V (a\vec{e}_1 + b\vec{e}_2 + c\vec{e}_3) \cdot d\vec{V} = 0$.

Ex. 1.1.1 In spherical coordinates, $\vec{r} = \frac{1}{\sqrt{a^2+b^2+c^2}}(a\vec{e}_1 + b\vec{e}_2 + c\vec{e}_3)$.

(a) $\vec{r} = \frac{1}{\sqrt{a^2+b^2+c^2}}(a\vec{e}_1 + b\vec{e}_2 + c\vec{e}_3)$, $\vec{r} \cdot \vec{r} = 1$
 $\vec{r} \cdot \vec{r} = \frac{1}{\sqrt{a^2+b^2+c^2}}(a\vec{e}_1 + b\vec{e}_2 + c\vec{e}_3) \cdot (a\vec{e}_1 + b\vec{e}_2 + c\vec{e}_3) = 1$

(b) $\vec{r} = \frac{1}{\sqrt{a^2+b^2+c^2}}(a\vec{e}_1 + b\vec{e}_2 + c\vec{e}_3)$, $\vec{r} \cdot \vec{r} = 1$
 $\vec{r} \cdot \vec{r} = \frac{1}{\sqrt{a^2+b^2+c^2}}(a\vec{e}_1 + b\vec{e}_2 + c\vec{e}_3) \cdot (a\vec{e}_1 + b\vec{e}_2 + c\vec{e}_3) = 1$

Ex. 1.1.2 For radial vector $\vec{r} = a\vec{e}_1$, $\vec{r} \cdot \vec{r} = \frac{1}{\sqrt{a^2+b^2+c^2}}(a\vec{e}_1 + b\vec{e}_2 + c\vec{e}_3) \cdot (a\vec{e}_1 + b\vec{e}_2 + c\vec{e}_3) = 1$
 Using divergence theorem, we have

$\frac{1}{\sqrt{a^2+b^2+c^2}} \int_V \vec{r} \cdot d\vec{V} = \frac{1}{\sqrt{a^2+b^2+c^2}} \int_V (\vec{r} \cdot \vec{r}) \cdot d\vec{V} = \frac{1}{\sqrt{a^2+b^2+c^2}} \int_V 1 \cdot d\vec{V} = 1$.

Ex. 1.1.3 $\vec{r} \cdot \vec{r} = (\int_{\vec{e}_1} + \int_{\vec{e}_2} + \int_{\vec{e}_3}) \cdot \vec{r} \cdot d\vec{V}$

$$\text{For } \text{face } (z=4): \vec{A} = A_z \hat{z} = A_z \hat{z}, \quad d\vec{A} = A_z d\vec{a}.$$

$$\int_{\text{face}} \vec{B} \cdot d\vec{A} = \int_{\text{face}} B_z d\vec{a} = B_z (\pi r^2) = 200\pi.$$

$$\text{For face } (z=0): \vec{A} = A_z \hat{z} = A_z \hat{z}, \quad d\vec{A} = -A_z d\vec{a}.$$

$$\int_{\text{face}} \vec{B} \cdot d\vec{A} = 0.$$

$$\text{For face } (r=R): \vec{A} = A_r \hat{r} = A_r \hat{r}, \quad d\vec{A} = A_r d\vec{a}.$$

$$\int_{\text{face}} \vec{B} \cdot d\vec{A} = 2B \int_{\text{face}} d\vec{a} = 2B (2\pi R \cdot 4) = 1600\pi.$$

$$\therefore \oint_C \vec{B} \cdot d\vec{A} = 200\pi + 0 + 1600\pi = 1800\pi.$$

$$\Phi \cdot \vec{B} = (1+z), \quad \int_V \Phi \cdot \vec{B} d\tau = \int_0^4 \int_0^{2\pi} \int_0^R \Phi \cdot \vec{B} \cdot d\vec{a} dr = 1600\pi \\ = \oint_C \vec{B} \cdot d\vec{A}.$$

$$\text{E1-18} \quad \Phi \cdot \vec{B} = \frac{1}{2} \frac{B}{r^2} (r^2) = \frac{1}{2} B_z = b_z.$$

$$\int_V \Phi \cdot \vec{B} d\tau = b_z V = b_z (\pi r^2 \cdot 4) = 24\pi b_z = \oint_C \vec{B} \cdot d\vec{A}.$$

Strangely, we haven't been told that $\vec{B}$ has a singularity inside the volume, but $r=0$.

$$\text{E1-21} \quad \text{For } (z=2R): \quad \vec{B} \cdot \vec{A} = \lim_{\epsilon \rightarrow 0} \frac{\int_V \vec{A} \cdot d\vec{A}}{\pi \epsilon^2}. \quad (1)$$

Referring to Fig. E1-21, we note that the areas on the opposite sides of a differential volume in cylindrical coordinates are the same in the r - and ϕ -directions, but are different in the z -direction. Let us first apply the continuity to $\oint_C \vec{B} \cdot d\vec{A}$ of the inside cylinder, then on the outside face:

$$\begin{aligned} \int_{\text{open face}} \vec{B} \cdot d\vec{A} &= \lim_{\epsilon \rightarrow 0} \int_{\text{open face}} \vec{A} \cdot d\vec{A} = A_z (b_1 - \Phi(b_1, b_2)) (b_2 - \Phi) \pi \epsilon^2 \\ &= \left[A_z (b_1, b_2) - \Phi \frac{\partial A_z}{\partial r} \right]_{r=b_1} (b_2 - \Phi) \pi \epsilon^2. \end{aligned} \quad (2)$$

On the outside face-

$$\begin{aligned}\int_{\text{outside}} \vec{A} \cdot d\vec{l} &= A_r \left(1 + \frac{1}{2} \epsilon_0 \mu_0 \frac{1}{r^2} \right) \int_{\text{outside}} r^2 dr \\ &= \left(A_r (r_0, \theta, \phi) + \frac{1}{2} \frac{dA_r}{dr} \right) \int_{r_0, \theta, \phi}^{\infty} r^2 dr.\end{aligned}\quad (3)$$

Adding (2) and (3) we have

$$\begin{aligned}\left(\int_{\text{inside}} + \int_{\text{outside}} \right) \vec{A} \cdot d\vec{l} &= \left(A_r + \epsilon_0 \mu_0 \frac{1}{2r^2} \right) \int_{r_0, \theta, \phi}^{\infty} r^2 dr + K \pi r_0 \\ &= \frac{1}{2\pi} \oint_{\text{circle}} (r \cdot A_r) \left(\int_{r_0, \theta, \phi}^{\infty} dr \right) + K \pi r_0.\end{aligned}\quad (4)$$

where R.H.T. contains integral around and higher powers of $r\theta$.
The sum of the contribution of the front and back faces (differential area = $dr \cdot d\theta \cdot r$) is

$$\left(\int_{\text{front}} + \int_{\text{back}} \right) \vec{A} \cdot d\vec{l} = \frac{1}{2\pi} \oint_{\text{circle}} \left(dr \cdot d\theta \cdot r \right) + K \pi r_0.\quad (5)$$

where R.H.T. contains integral around and higher powers of $r\theta$.
Similarly, the sum of the contribution of the top and bottom faces (differential area = $r \cdot dr \cdot d\theta$) is

$$\left(\int_{\text{top}} + \int_{\text{bottom}} \right) \vec{A} \cdot d\vec{l} = \left(r \cdot \frac{1}{2\pi} \frac{dA_r}{dr} \right) \int_{r_0, \theta, \phi}^{\infty} dr \cdot d\theta + K \pi r_0.\quad (6)$$

where R.H.T. contains integral around and higher powers of $r\theta$.

Combining (2), (3) and (6) in (4), dividing by the separating differential setting $dr \cdot d\theta \cdot d\phi = 0$, we get

$$\vec{F} \cdot \vec{A} = \frac{1}{2\pi} \oint_{\text{circle}} (r \cdot A_r) + \frac{1}{2} \frac{dA_r}{dr} + \frac{K}{2\pi}.$$

where the subscript θ has been dropped for simplicity.

Ex. 2.1.13 a) $\vec{F} = x_2 \frac{\partial}{\partial x_1} \hat{e}_1$, $ds = x_2^2 \sin \theta \, d\theta \, d\varphi$.

$$\oint_C \vec{F} \cdot d\vec{s} = \int_0^{2\pi} \int_0^\pi (x_2 - \varphi) \sin \theta \, d\theta \, d\varphi = -\pi.$$

b) $\vec{F} \cdot \vec{S} = -\frac{\partial}{\partial x_1} \left(\frac{x_2^3}{3} \right)$, $ds = x_2^2 \sin \theta \, d\theta \, d\varphi$.

$$\int_V \vec{F} \cdot \vec{S} \, dv = \int_0^{2\pi} \int_0^\pi \int_0^1 \left(-\frac{\partial}{\partial x_1} \left(\frac{x_2^3}{3} \right) \right) x_2^2 \sin \theta \, dx_1 \, d\theta \, d\varphi = -\pi.$$

Ex. 2.1.14 $\vec{F} = x_2 x_1 x_1^2 \hat{e}_1 - x_2^2 x_1^2 \hat{e}_2$; $d\vec{s} = x_2 dx_1 + dx_2 \hat{e}_2$.



a) $\vec{F} \cdot d\vec{s} = x_2^3 x_1^2 dx_1 - x_1^2 x_2^2 dx_2$

Path ①: $\rightarrow \pi$, $\int_0^1 dx_1 \int_0^1 x_2^3 x_1^2 dx_1 = \pi$

Path ②: $\rightarrow -1$, since $\int_0^1 dx_2 \int_0^1 x_1^2 x_2^2 dx_1 = 0$

Path ③: $\rightarrow -1$, since $\int_0^1 dx_2 \int_0^1 x_1^2 x_2^2 dx_1 = 0$

$$\therefore \oint_C \vec{F} \cdot d\vec{s} = \pi + \frac{\pi}{2} - \pi = \frac{\pi}{2} = \pi \frac{1}{2}$$

b) $\vec{F} \cdot \vec{S} = -x_2 \frac{\partial}{\partial x_1} (x_1^3) = -3x_1^2 x_2$, $ds = x_2 dx_1 dx_2$.

$$\int_V (\vec{F} \cdot \vec{S}) \, dv = -3 \int_0^1 dx_2 \int_0^1 x_1^2 dx_1 = -\pi \frac{1}{2}$$

Ex. 2.1.15 $\oint_C \vec{F} \cdot d\vec{s} = \lim_{\Delta x_2 \rightarrow 0} \frac{1}{\Delta x_2} \left(\oint_C \vec{F} \cdot d\vec{s} \right)$ ①



where Δx_2 is the length of ①

and the counter clockwise of the four sides (2,3,4)

side 1, $d\vec{s} = \Delta x_2 \hat{e}_1$.

$$\vec{F} \cdot d\vec{s} = x_2 \Delta x_2 \left(1 + \frac{\partial}{\partial x_1} \right) x_2$$

$$\text{where } x_2 \Delta x_2 \left(1 + \frac{\partial}{\partial x_1} \right) x_2 = \frac{\partial}{\partial x_1} \left(\frac{x_2^3}{3} \right) + x_2 \Delta x_2$$

$$\oint_C \vec{F} \cdot d\vec{s} = \left[x_2 - \frac{\partial}{\partial x_1} \left(\frac{x_2^3}{3} \right) \right]_{x_2, x_1} \Delta x_2$$

$$\begin{aligned} \text{Euler: } dL &= -\lambda_2 (d_2 dL), \quad dL = d_2 (d_2 dL + \frac{1}{2} d_2^2 dL) \\ \int_{t=0}^T dL &= - \left[\left(d_2 + \frac{1}{2} \frac{d_2^2}{d_2} \right) \right]_{t=0}^T d_2 dL + d_2 dL \quad \textcircled{3} \end{aligned}$$

Combining ② and ③:

$$\int_{t=0}^T dL = \left(-\frac{d_2^2}{2} + d_2 dL \right) \Big|_{t=0}^T d_2 dL \quad \textcircled{4}$$

$$\begin{aligned} \text{Euler: } dL &= d_2 d_2 + d_2 \frac{1}{2} d_2^2, \quad dL = d_2 (d_2 d_2 + \frac{1}{2} d_2^2 d_2 + d_2 \frac{1}{2} d_2^2) \\ \int_{t=0}^T dL &= \left[\frac{d_2^2}{2} + \frac{1}{2} \frac{d_2^3}{d_2} \right]_{t=0}^T + d_2 dL \Big|_{t=0}^T d_2 + d_2 \frac{1}{2} d_2^2 \Big|_{t=0}^T \quad \textcircled{5} \end{aligned}$$

$$\begin{aligned} \text{Euler: } \int_{t=0}^T dL &= \left[\left(d_2 + \frac{1}{2} \frac{d_2^2}{d_2} \right) + d_2 dL \right]_{t=0}^T d_2 dL + \frac{1}{2} d_2^2 \Big|_{t=0}^T \quad \textcircled{6} \end{aligned}$$

Combining ④ and ⑥:

$$\begin{aligned} \int_{t=0}^T dL &= \frac{d_2^2}{2} \Big|_{t=0}^T d_2 dL + d_2 dL \Big|_{t=0}^T d_2 dL + d_2 dL + d_2 dL \\ &= \frac{d_2}{2} (d_2 dL) \Big|_{t=0}^T d_2 dL + d_2 dL \quad \textcircled{7} \end{aligned}$$

Substituting ③, ④ and ⑤ in ⑦ we obtain:

$$C(P, T)_k = \frac{1}{2} \frac{d_2^2}{d_2^2} \left[\frac{d_2}{d_2} (d_2 dL) - \frac{d_2^2}{d_2^2} \right] =$$

where the subscript k has been dropped for simplicity.

$$\text{Euler: } P_k L = \frac{1}{2} \frac{d_2^2}{d_2^2} (d_2 dL + dL - d_2 dL + dL).$$

$$\int_0^T P_k dL = \int_0^T \frac{1}{2} \frac{d_2^2}{d_2^2} (P_k dL)_{t=0} + (d_2^2 dL + dL) = 0.$$

$$\int_0^T P_k dL = \int_0^T (P_k)_{t=0} \cdot d_2 dL + \int_0^T dL = 0.$$

$$\text{Euler: } P = d_2 (d_2 + dL) + d_2 (d_2 dL + \frac{1}{2} d_2^2) + d_2 (dL + \frac{1}{2} d_2^2).$$

4) P is constant $\rightarrow P = P = 0$.

$$\text{or } d_2 \left(\frac{d_2^2}{d_2^2} - \frac{d_2^2}{d_2^2} \right) + d_2 \left(\frac{d_2^2}{d_2^2} - \frac{d_2^2}{d_2^2} \right) + d_2 \left(\frac{d_2^2}{d_2^2} - \frac{d_2^2}{d_2^2} \right) = 0.$$

which gives three equations:

$$\frac{\partial^2}{\partial x^2}(x + a_3 y + a_4 z) + \frac{\partial^2}{\partial y^2}(a_3 x + xz) = 0 \implies a_3 + 2a_4 = 0 \implies a_4 = -\frac{a_3}{2}$$

$$\frac{\partial^2}{\partial x^2}(x + a_4 z) + \frac{\partial^2}{\partial y^2}(x + a_3 y + a_4 z) = 0 \implies a_4 + 1 = 0 \implies a_4 = -1,$$

$$\frac{\partial^2}{\partial x^2}(a_3 x + xz) + \frac{\partial^2}{\partial y^2}(x + a_4 z) = 0 \implies a_3 = 2.$$

b) $\vec{F}$ is a conservative vector field $\implies \vec{F} \cdot \vec{F} = 0$,

$$\text{or} \quad \frac{\partial^2 F_x}{\partial x^2} + \frac{\partial^2 F_x}{\partial y^2} + \frac{\partial^2 F_x}{\partial z^2} = 0,$$

$$\text{or} \quad \frac{\partial^2}{\partial x^2}(x + a_4 z) + \frac{\partial^2}{\partial y^2}(a_3 x + xz) + \frac{\partial^2}{\partial z^2}(x + a_3 y + a_4 z) = 0,$$

$$\text{or} \quad 1 + a_4 = 0 \implies a_4 = -1.$$

c) $\vec{F} = -\vec{\nabla}V \implies \nabla_x(x + z) = \nabla_y(xz) = \nabla_z(x - yz + z)$

$$= -a_x \frac{\partial V}{\partial x} = -a_y \frac{\partial V}{\partial y} = -a_z \frac{\partial V}{\partial z}.$$

$$\frac{\partial V}{\partial x} = -(x + z) \implies V = -\frac{x^2}{2} - xz + f(y, z)$$

$$\frac{\partial V}{\partial y} = 1z \implies V = 1yz + f(y, z).$$

$$-\frac{\partial V}{\partial z} = -x + 1y + z \implies V = -xz + 1yz + \frac{z^2}{2} + f(y, z).$$

$$\therefore V = -\frac{x^2}{2} - xz + 1yz + \frac{z^2}{2}.$$

Chapter 4

Solution of Electrostatic Problems

Ex-1 Use subscripts 1 and 2 to denote dielectrics and air regions respectively. $\nabla^2 V = 0$ in both regions.

$$V_1 = a_1 y + a_2, \quad E_1 = -a_1 \hat{y}, \quad D_1 = -\epsilon_1 a_1 \hat{y}$$

$$V_2 = a_3 y + a_4, \quad E_2 = -a_3 \hat{y}, \quad D_2 = -\epsilon_2 a_3 \hat{y}$$

$$\text{BC: at } y=0, \quad V_1 = V_2, \quad \text{at } y=a, \quad V_2 = V_3$$

$$\text{at } y=0 \text{ and } a: \quad V_2 = V_3, \quad E_2 = E_3$$

$$\text{Solving: } a_1 = \frac{V_1}{a}, \quad a_2 = 0, \quad a_3 = \frac{V_2}{a}, \quad a_4 = \frac{V_2}{a}$$

$$\text{a) } V_1 = \frac{V_1}{a} y, \quad E_1 = -\frac{V_1}{a} \hat{y}$$

$$\text{b) } V_2 = \frac{V_2}{a} y, \quad E_2 = -\frac{V_2}{a} \hat{y}$$

$$\text{c) } (D_1)_y = -\epsilon_1 \frac{V_1}{a}, \quad (D_2)_y = -\epsilon_2 \frac{V_2}{a}$$

$$(D_1)_y = (D_2)_y = -\frac{\epsilon_1 V_1}{a}$$

Ex-2 At a point where V is a maximum (or minimum) second derivatives of V with respect to x, y , and z must all be negative (simultaneously) that can never occur, as required by Laplace's equation.

Ex-3 Potential eq. $\nabla^2 V = -\frac{\rho}{\epsilon_0} \rightarrow \frac{1}{r} \frac{d}{dr} \left(r \frac{dV}{dr} \right) = -\frac{\rho}{\epsilon_0}$

Solution: $V = -\frac{\rho}{4\epsilon_0} r^2 + a_1 r + a_2$

BC: for $r=a$, $V = -\frac{\rho}{4\epsilon_0} a^2 + a_1 a + a_2, \quad a = \frac{V(a) + \frac{\rho a^2}{4\epsilon_0}}{a}$

for $r=b$, $V = -\frac{\rho}{4\epsilon_0} b^2 + a_1 b + a_2, \quad a = \frac{V(b) + \frac{\rho b^2}{4\epsilon_0}}{b}$



$$E \Big|_{r=a} = \epsilon_1 \frac{V}{\ln(b/a)} \hat{r} = -\epsilon_1 \frac{V}{\ln(b/a)} \hat{r}$$

$$\text{a) } E = E_r = E \Big|_{r=a} = -\frac{\epsilon_1 V}{\ln(b/a)} \hat{r}$$

$$\text{b) } \int_a^b E_r dr = -V$$

Ex 4-8



Calculate the conditions in the xy -plane ($z=0$).

a) $V = \frac{1}{2} \log \left(\frac{1}{x} - \frac{1}{x} + \frac{1}{y} - \frac{1}{y} \right)$, where

$$x_1 = [(x-a)^2 + (y-b)^2]^{1/2},$$

$$x_2 = [(x-a)^2 + (y-b)^2]^{1/2},$$

$$x_3 = [(x-a)^2 + (y-b)^2]^{1/2},$$

$$x_4 = [(x-a)^2 + (y-b)^2]^{1/2}.$$

$$E_1 = -\frac{1}{2} V_1 = -E_2 \frac{\partial V}{\partial x} - E_3 \frac{\partial V}{\partial y}$$

$$= -E_2 \frac{\partial}{\partial x} \left[-\frac{1}{x_1^2} + \frac{1}{x_2^2} - \frac{1}{x_3^2} + \frac{1}{x_4^2} \right]$$

$$+ E_3 \frac{\partial}{\partial y} \left[-\frac{1}{x_1^2} + \frac{1}{x_2^2} - \frac{1}{x_3^2} + \frac{1}{x_4^2} \right].$$

E_2 will have a discontinuity at the point P located in the xy -plane.

b) On the vanishing field planes, $E_1 = E_2$ and E_3 .

Along the x -axis, give $E_1 = [(x-a)^2 + b^2]^{1/2} = x_1$,

and $E_2 = [(x-a)^2 + a^2]^{1/2} = x_2$.

$$E_3 = 0, \quad E_4 = \frac{1}{2} \log \left(\frac{1}{x} - \frac{1}{x} \right)$$

$$\therefore E(x=0) = \frac{1}{2} \log \left[\frac{1}{[(x-a)^2 + b^2]^{1/2}} - \frac{1}{[(x-a)^2 + a^2]^{1/2}} \right]$$

$$= 0, \quad \text{at } x=0,$$

$$\text{non-0, at } x=a.$$

Similarly for $E(x=a)$ on the vanishing vanishing planes by changing a to x and b to a .

Ex 4-9 Refer to Example 4-4

$$C = \frac{\frac{1}{2} \log \left[\frac{1}{[(x-a)^2 + b^2]^{1/2}} - \frac{1}{[(x-a)^2 + a^2]^{1/2}} \right]}{\frac{1}{2} \log \left[\frac{1}{[(x-a)^2 + b^2]^{1/2}} - \frac{1}{[(x-a)^2 + a^2]^{1/2}} \right]} \quad (C(x))$$

Ex. 11.1 Same as C_{12} in problem 11.9-12.

Ex. 11.2 a) From Eqs. (9-48) and (9-49):

$$V_p = \frac{Q_p}{4\pi\epsilon_0} \ln \left[\frac{4\pi\epsilon_0 R_p}{R_p - R_p'} \right]$$

$$E_p = -\hat{r}_p \frac{1}{R_p} \frac{Q_p}{4\pi\epsilon_0} - \hat{r}_p' \frac{Q_p}{R_p'} = -\frac{Q_p}{4\pi\epsilon_0} \left[\frac{\hat{r}_p}{R_p(R_p - R_p')} + \frac{\hat{r}_p'}{R_p'(R_p - R_p')} \right]$$

b) Equation for line everywhere tangent to the electric field lines is obtained by requiring

$$\frac{dy}{dx} = \frac{E_y}{E_x} = - \frac{dy/dy}{dx/dx}$$

which reduces to $\frac{dy/dy}{dx/dx} = \frac{dy}{x}$.

Integrating, we obtain $x^2 + y^2 = 2ky + c$,

$$\text{or } x^2 + (y - k)^2 = (k^2 + c)$$

where k is a constant. Circles of radii $\sqrt{k^2 + c}$ having centers at $(0, k)$.



$$V_1 = \frac{Q_1}{4\pi\epsilon_0} \ln \frac{2}{a_1}, \quad V_2 = \frac{Q_2}{4\pi\epsilon_0} \ln \frac{2}{a_2}$$

Capacitance per unit length

$$C' = \frac{Q_1}{V_1 - V_2} = \frac{4\pi\epsilon_0}{\ln \frac{a_2}{a_1}}$$

For separation:

$$a_1' = a_1, \quad a_2' = a_2, \quad d_1' = d_2, \quad d_2' = d_1$$

$$d_1 = a_1 + d, \quad d_2 = a_2 + d$$

Let us obtain

$$\frac{d_1 d_2}{a_1 a_2} = \frac{d_1' d_2'}{a_1' a_2'} \quad \text{and} \quad a_1' = a_1' + d, \quad a_2' = a_2' + d$$

$$\frac{d_1 d_2}{a_1 a_2} = \frac{d_1' d_2'}{a_1' a_2'} = \frac{d_1' d_2'}{a_1' a_2' + d(a_1' + a_2') + d^2}$$

$$\therefore C' = \frac{4\pi\epsilon_0}{\ln \left[\frac{d_1 d_2}{a_1 a_2} \frac{a_1' a_2' + d(a_1' + a_2') + d^2}{d_1' d_2'} \right]}$$

$$= \frac{4\pi\epsilon_0}{\ln \left[\frac{d_1 d_2}{a_1 a_2} \frac{a_1' a_2' + d(a_1' + a_2') + d^2}{d_1' d_2'} \right]} \quad (P. 11.1)$$

Ex. 4-13 $d_p = 1 + \cos \theta$, $v_1 = \frac{1}{\sqrt{1-\cos^2 \theta}} (v_1' - v_1' \cos^2 \theta)$, $d_p = 1 + \cos \theta$, $v_2 = \frac{1}{\sqrt{1-\cos^2 \theta}} (v_2' - v_1' \cos^2 \theta)$



$d_p = 1 + \cos \theta$, $v_1' = v_1' - v_1' \cos^2 \theta$

$d_p = 1 + \cos \theta$, $v_2' = v_2' - v_1' \cos^2 \theta$

(a) $V = \frac{1}{\sqrt{1-\cos^2 \theta}} \ln \left[\frac{1 + \cos \theta}{1 - \cos \theta} \right] \ln \left[\frac{1 + \cos \theta}{1 - \cos \theta} \right]$

At $\theta = 0$: $v_1 = 1 + \cos \theta = 2$, $v_2 = 1 + \cos \theta = 2$

At $\theta = \pi$: $v_1 = 1 + \cos \theta = 0$, $v_2 = 1 + \cos \theta = 0$

$V = \frac{1}{\sqrt{1-\cos^2 \theta}} \ln \left[\frac{1 + \cos \theta}{1 - \cos \theta} \right] \ln \left[\frac{1 + \cos \theta}{1 - \cos \theta} \right]$

Expressing V in terms of θ and $\cos \theta$

and simplifying: $V = \frac{1}{\sqrt{1-\cos^2 \theta}} \ln \left[\left(\frac{1 + \cos \theta}{1 - \cos \theta} \right)^2 \right] \ln \left[\left(\frac{1 + \cos \theta}{1 - \cos \theta} \right)^2 \right]$

$V = \frac{1}{\sqrt{1-\cos^2 \theta}} \ln \left[\left(\frac{1 + \cos \theta}{1 - \cos \theta} \right)^2 \right] \ln \left[\left(\frac{1 + \cos \theta}{1 - \cos \theta} \right)^2 \right]$ (104)

(b) Force per unit length $F = \frac{1}{\sqrt{1-\cos^2 \theta}} \ln \left[\left(\frac{1 + \cos \theta}{1 - \cos \theta} \right)^2 \right] \ln \left[\left(\frac{1 + \cos \theta}{1 - \cos \theta} \right)^2 \right]$ (105)

Ex. 4-14



$d_p = \frac{1}{\sqrt{1-\cos^2 \theta}}$, $v_1 = \frac{1}{\sqrt{1-\cos^2 \theta}}$

(a) $V = \frac{1}{\sqrt{1-\cos^2 \theta}} \ln \left[\frac{1 + \cos \theta}{1 - \cos \theta} \right]$

$v_1 = \frac{1}{\sqrt{1-\cos^2 \theta}} \ln \left[\frac{1 + \cos \theta}{1 - \cos \theta} \right]$

$v_2 = \frac{1}{\sqrt{1-\cos^2 \theta}} \ln \left[\frac{1 + \cos \theta}{1 - \cos \theta} \right]$

(b) $F = \frac{1}{\sqrt{1-\cos^2 \theta}} \ln \left[\frac{1 + \cos \theta}{1 - \cos \theta} \right] \ln \left[\frac{1 + \cos \theta}{1 - \cos \theta} \right]$

Ex. 4-15 (See next page.)

Ex. 4-16 The potential boundary conditions at end 1: $V = V_1$, and $\frac{1}{\sqrt{1-\cos^2 \theta}} \ln \left[\frac{1 + \cos \theta}{1 - \cos \theta} \right]$

From Fig. 4-13 and the d_p pathlengths from (a) and (b)

$V = \frac{1}{\sqrt{1-\cos^2 \theta}} \ln \left[\frac{1 + \cos \theta}{1 - \cos \theta} \right] \ln \left[\frac{1 + \cos \theta}{1 - \cos \theta} \right]$

$V = \frac{1}{\sqrt{1-\cos^2 \theta}} \ln \left[\frac{1 + \cos \theta}{1 - \cos \theta} \right] \ln \left[\frac{1 + \cos \theta}{1 - \cos \theta} \right]$

To obtain the velocity the d_p 's are used, we require

$\frac{1}{\sqrt{1-\cos^2 \theta}} \ln \left[\frac{1 + \cos \theta}{1 - \cos \theta} \right]$ and $d_p = d_1 + d_2 \rightarrow d_1 = d_2 = \frac{1}{\sqrt{1-\cos^2 \theta}}$

Ex-56



a) $\vec{E}_2$ and system of image charges?

In left sphere

$$E_2 \text{ at } d_1 = 2$$

$$q_1 = \frac{4\pi\epsilon_0 a^2}{d_1^2} E_2 \text{ at } d_1$$

$$q_1 = \frac{4\pi\epsilon_0 a^2}{d_1^2} E_2$$

$$q_2 = \frac{4\pi\epsilon_0 a^2}{d_2^2} E_2 \text{ at } d_2$$

$$d_1 = \frac{a}{\cos \theta_1}, d_2 = \frac{a}{\cos \theta_2}$$

$$b) C = \frac{4\pi\epsilon_0 a^2}{d_1^2} E_2 + \frac{4\pi\epsilon_0 a^2}{d_2^2} E_2$$

Ex-57 a) $V(x,y,z)$ Required boundary conditions



① $V(x,y,z) = 0$ — Grounded conducting plane at $x=0$

② $V(x,y,z) = 0$ — Grounded conducting plane at $x=a$

③ $V(x,y,z) = V_0 \cos \theta$ — $V_0 \cos \theta$ — $V_0 \cos \theta$

④ Shape of upper boundary defined by $V(x,y,z) = V_0 \cos \theta$ or $V_0 \cos \theta$ (hyperbolic)



b) $V(x,y,z)$ Required boundary conditions

① $V(x,y,z) = 0$ — Grounded plane at $x=0$

② $V(x,y,z) = 0$ — Grounded plane at $x=a$

③ $V(x,y,z) = V_0 \cos \theta$ — $V_0 \cos \theta$ — $V_0 \cos \theta$

④ Shape of upper boundary defined by $V(x,y,z) = V_0 \cos \theta$ or $V_0 \cos \theta$ (hyperbolic)

⑤ $V(x,y,z) = V_0 \cos \theta$ — $V_0 \cos \theta$ — $V_0 \cos \theta$

Ex-16



a) Q_1 and system of image charges:

In left sphere

q_1 at $d_1 < 0$

$q_2 = \frac{a}{d_1} q_1$ at d_2

$q_3 = \frac{a}{d_2} q_2$ at d_3

$\vdots$

$q_n = q_1 \left(\frac{a}{d_1} \right)^n$ at d_n $q_{n+1} = q_2 \left(\frac{a}{d_2} \right)^n$ at d_{n+1}

$(d_1, d_2, \dots) \rightarrow 0$ $d_n = \frac{a^2}{d_{n-1}}$ $\rightarrow (d_1, d_2, \dots) \rightarrow d_1 < 0$

In right sphere

$-q_2 = \frac{a}{d_2} q_1$ at d_1

$-q_3 = \frac{a}{d_1} q_2 = \frac{a^2}{d_1 d_2} q_1$ at d_2

$\vdots$

$-q_n = q_2 \left(\frac{a}{d_2} \right)^n$ at d_n $-q_{n+1} = q_1 \left(\frac{a}{d_1} \right)^n$ at d_{n+1}

$$\text{so } Q = \sum_{n=1}^{\infty} q_n = q_1 \left[1 + \sum_{n=1}^{\infty} \left(\frac{a}{d_1} \right)^n \right]$$

Ex-17 a) $V(x, y) = 0$ Required boundary conditions:



① $V(x, y) = 0$ — grounded conducting plane at $y=0$

② $V(x, y) = 0$ — grounded conducting plane at $x=0$

or $V(x, y) = 0$ — $x = \frac{a}{2}$ — equipotential $\frac{a}{2}$

③ Shape of second boundary surface:

Charge q at $(\frac{a}{2}, 0)$ or $(0, \frac{a}{2})$ — potential is symmetrical

at $V(x, y) = 0$ plane only: required boundary condition:

① $V(x, y) = 0$ — grounded plane at $y=0$

② equipotential — grounded plane at $x = \frac{a}{2}$

③ equipotential — grounded plane at $y = \frac{a}{2}$

④ $V(x, y) = 0$ — $x = \frac{a}{2}$ — equipotential $\frac{a}{2}$

⑤ $V(x, y) = 0$ — $y = \frac{a}{2}$ — equipotential $\frac{a}{2}$



⑥ $V(x, y) = 0$ — $x = \frac{a}{2}$ — equipotential $\frac{a}{2}$ and $y = \frac{a}{2}$ — equipotential $\frac{a}{2}$

$$\underline{E.4.22} \quad V(x, y) = C_0 \sinh \frac{\pi y}{b} (\alpha - x) \cos \frac{\pi x}{b},$$

$$\underline{E.4.23} \quad V_0(x, y) = C_{0n} \sinh \frac{\pi y}{b} \sin \frac{\pi x}{b},$$

$$\sum_{n=1}^{\infty} C_{0n} V_0 = \sum_{n=1}^{\infty} C_{0n} V(x, y) = \sum_{n=1}^{\infty} C_{0n} \sinh \frac{\pi y}{b} \sin \frac{\pi x}{b},$$

$$V(x, y) = \frac{4V_0}{\pi} \sum_{n=1}^{\infty} \frac{\sinh \frac{\pi y}{b} \sin \frac{\pi x}{b}}{\sinh \frac{\pi y_0}{b} \sin \frac{\pi x_0}{b}} \sin \frac{\pi x}{b}.$$

$$\underline{E.4.24} \quad V(x, y) = \frac{V_0}{2} \sin \frac{\pi y}{b} \left[J_0 \sinh \frac{\pi x}{b} + J_2 \sinh \frac{\pi x}{b} \right],$$

$$\text{At } y=0, \quad V(x, 0) = V_0 = \frac{V_0}{2} J_0 \sin \frac{\pi x}{b} \implies J_0 = \left\{ \frac{2V_0}{\sinh \frac{\pi x_0}{b}} \right\},$$

$$\begin{aligned} \text{At } y=b, \quad V(x, b) = V_0 &= \frac{V_0}{2} \sin \frac{\pi y}{b} \left[J_0 \sinh \frac{\pi x}{b} + J_2 \sinh \frac{\pi x}{b} \right] \\ &\implies J_2 \sinh \frac{\pi x}{b} = J_0 \sinh \frac{\pi x}{b} \implies J_2 = \left\{ \frac{2V_0}{\sinh \frac{\pi x_0}{b}} \right\} \\ \therefore J_2 &= \left\{ \frac{\sinh \frac{\pi x_0}{b}}{\sinh \frac{\pi x_0}{b}} \right\} (J_0 - J_0 \sinh \frac{\pi x_0}{b}), \quad \text{where } \frac{J_2}{J_0} = \frac{1}{\sinh \frac{\pi x_0}{b}}. \end{aligned}$$

$$\underline{E.4.25} \quad V(x, y) = \sum_n \sum_m C_{nm} \sin \frac{\pi m y}{b} \sin \frac{\pi n x}{b} \sinh k_{nm} x,$$

$$\text{where } k_{nm} = \sqrt{\left(\frac{\pi m}{b}\right)^2 + \left(\frac{\pi n}{a}\right)^2}.$$

$$\begin{aligned} \text{At } x=0, \quad V(x, 0) = V_0 &= \sum_n \sum_m C_{nm} \sin \frac{\pi m y}{b} \sin \frac{\pi n x}{b} \sinh k_{nm} x \\ &\implies C_{nm} = \left\{ \frac{2V_0 \sinh \frac{\pi x_0}{b}}{\sinh \frac{\pi x_0}{b} \sinh k_{nm} x_0} \right\}, \quad \text{where } \frac{C_{nm}}{V_0} = \frac{2}{\sinh \frac{\pi x_0}{b} \sinh k_{nm} x_0}. \end{aligned}$$

$$\underline{E.4.26} \quad \text{Solutions: } V(x) = J_1 x + J_2.$$

$$\text{a) E.C. (i): } V(x) = 0 \implies J_1 x + J_2 = 0.$$

$$\text{E.C. (ii): } V(x) = V_0 = J_1 x + J_2 \implies J_1 = \frac{V_0}{x_0} \left. \vphantom{\begin{matrix} J_1 \\ J_2 \end{matrix}} \right\} \therefore V(x) = \frac{V_0}{x_0} x.$$

$$\text{b) E.C. (i): } V(x) = V_0 = J_1 x + J_2 \left. \vphantom{\begin{matrix} J_1 \\ J_2 \end{matrix}} \right\} \implies J_1 = -\frac{V_0}{x_0}, \quad J_2 = \frac{V_0 x_0}{x_0}.$$

$$\text{E.C. (ii): } V(x) = 0 = J_1 x + J_2 \left. \vphantom{\begin{matrix} J_1 \\ J_2 \end{matrix}} \right\} \implies J_1 = -\frac{V_0}{x_0}, \quad J_2 = \frac{V_0 x_0}{x_0}.$$

$$\therefore V(x) = \frac{V_0}{x_0} (x_0 - x), \quad 0 \leq x \leq x_0.$$

Chapter 3

Steady Electric Currents

Ex-1 a) Integrating E_r (Gauss) $V(r) = \left[\frac{E_0}{2\epsilon_0} r^2 \right] r^{2n} = \frac{E_0}{2\epsilon_0} r^{2n+2}$
 $E(r) = -\frac{1}{r} \frac{dV}{dr} = -\frac{E_0}{2\epsilon_0} \frac{d}{dr} \left(\frac{r^{2n+2}}{2n+2} \right)$ (1)

b) $\rho(r) = \epsilon_0 \frac{dE_r}{dr} = -\frac{E_0}{2\epsilon_0} \frac{d}{dr} \left(\frac{r^{2n+2}}{2n+2} \right)$

$$Q = \int_0^d \rho(r) 2\pi r dr = -\frac{E_0}{2\epsilon_0} \frac{d}{dr} \left(\frac{r^{2n+2}}{2n+2} \right) \int_0^d r^{2n+1} dr = -\frac{E_0}{2\epsilon_0} \frac{d}{dr} \left(\frac{r^{2n+2}}{2n+2} \right) \frac{r^{2n+2}}{2n+2}$$

c) On the inside, $r=d$, $E_r = -\frac{1}{2} E_0 d^{2n} = -\frac{E_0 d^{2n}}{2}$

Total surface charge on inside $= E_r d = -\frac{E_0 d^{2n+1}}{2}$

Total charge on outside $= 0$

d) Substituting (1) in E_r (Gauss)

$$E_r = \frac{E_0}{2\epsilon_0} - \left(\frac{E_0}{2\epsilon_0} r \right)^{2n} \left(\frac{r}{r} \right)^{2n}, \quad r^{2n+2} d_r = \left(\frac{E_0}{2\epsilon_0} \frac{r^{2n+2}}{2n+2} \right) dr$$

Integrating: $r = \left(\frac{2\epsilon_0}{E_0} \frac{2n+2}{2n+2} \right)^{1/(2n+2)} dr$

$\therefore$ From (1) $E_r = \frac{E_0}{2\epsilon_0} \left(\frac{r}{d} \right)^{2n}$

For V inside (a), $d = d^{2n+2}$ and E_r (Gauss) $E_r d = -\frac{E_0 d^{2n+1}}{2}$

and $d = d^{2n+2}$ (a), $E_r = \frac{E_0 d^{2n+2}}{2\epsilon_0} = \frac{E_0 d^{2n+2}}{2\epsilon_0}$

Ex-2 $R_1 =$ Resistance per unit length of wire $= \frac{\rho}{A_1} = \frac{\rho}{\pi r_1^2}$

$R_2 =$ Resistance per unit length of coating $= \frac{\rho}{\pi r_2^2}$

Let $k =$ Thickness of coating, then $r_2 = r_1 + k$ (a) (b) (c) (d) (e)

a) $R_1 = R_2 \implies \frac{\rho}{\pi r_1^2} = \frac{\rho}{\pi r_2^2} \implies r_1 = r_2$

b) $R_1 = R_2 = \frac{\rho}{\pi r_1^2} \implies \frac{\rho}{\pi r_1^2} = \frac{\rho}{\pi r_2^2} \implies \frac{\rho}{\pi r_1^2} = \frac{\rho}{\pi r_2^2} \implies \frac{\rho}{\pi r_1^2} = \frac{\rho}{\pi r_2^2}$

$\frac{\rho}{\pi r_1^2} = \frac{\rho}{\pi r_2^2} \implies \frac{\rho}{\pi r_1^2} = \frac{\rho}{\pi r_2^2} \implies \frac{\rho}{\pi r_1^2} = \frac{\rho}{\pi r_2^2}$

Thus, $r_1 = r_2$ and $R_1 = R_2$

Ex. 2.11 a) $E_y(z=0^+)=E_{y0}+E_{yR} \longrightarrow E_y \sin \alpha_1 = E_y \sin \alpha_2$
 $E_x(z=0^+)=E_{x0}+E_{xR} \longrightarrow E_y E_{y0}+E_y E_{yR} = E_y \sin \alpha_1 \sin \alpha_2 + E_y \sin \alpha_2 \sin \alpha_1$
 $\therefore E_{yR} = E_y \sqrt{\epsilon_2/\epsilon_1} \sin \alpha_2 = \left(\frac{\epsilon_2}{\epsilon_1}\right) E_y \sin \alpha_2$ (1)

$E_{y0} \sin \alpha_1 = \left(\frac{\epsilon_2}{\epsilon_1}\right) E_{y0} \sin \alpha_2 \longrightarrow \alpha_2 = \sin^{-1} \left(\frac{\epsilon_1}{\epsilon_2} \sin \alpha_1\right)$ (2)

b) $E_y(z=0^+)=E_{y0}-E_{yR}=E_y \longrightarrow E_y E_{y0}-E_y E_{yR}=E_y$

$E_{yR} = \left(\frac{\epsilon_2}{\epsilon_1}\right) E_{y0} - E_y = \left(\frac{\epsilon_2}{\epsilon_1}\right) E_y - E_y \sin \alpha_1$

c) IF both media are perfect dielectrics, $\epsilon_1 = \epsilon_2 = \epsilon$, E_{y0}

(1) and (2) reduce to $E_y(z=0^+)=E_y(z=0^-)$ respectively and $\alpha_2 = \alpha_1$

Ex. 2.12



$E_{y1} = E = (V/d) \frac{d}{2}$

a) The electric field in region 1 and 2 according to the above diagram

$E = E_1 E_2 \longrightarrow E = \frac{V}{d} = E_1 \frac{d}{2}$

$E_2 = \left(\frac{2}{d}\right) E_1 d = \left(\frac{2}{d} \frac{V}{2}\right) \frac{d}{2} = \frac{V}{d} = E$

$E = \frac{V}{d} = \frac{E_1}{2} = \frac{E_2}{2} = \frac{V}{2d} \Rightarrow E = \frac{V}{d}$

b) $Q_1 = E_1 \epsilon_1 A = \frac{V}{2d} \epsilon_1 A = \frac{V \epsilon_1 A}{2d}$ on upper plate

$Q_2 = E_2 \epsilon_2 A = \frac{V}{2d} \epsilon_2 A = \frac{V \epsilon_2 A}{2d}$ on lower plate

c) $Q = Q_1 + Q_2 = \frac{V}{2d} \epsilon_1 A + \frac{V}{2d} \epsilon_2 A = \frac{V}{2d} A (\epsilon_1 + \epsilon_2) = \frac{V}{2d} A \frac{2\epsilon_1 \epsilon_2}{\epsilon_1 + \epsilon_2}$

Ex. 2.13



a) $i_1 = \frac{V}{R_1} \quad i_2 = \frac{V}{R_2}$

$i_3 = \frac{V}{R_1} \quad i_4 = \frac{V}{R_2}$

b) $P = i_1^2 R_1 = i_2^2 R_2$

$= \frac{V^2}{R_1} = \frac{V^2}{R_2}$

Ex. 2.14 Refer to Fig. 2.4. In the transient state, the equation of continuity must be satisfied at the interface.

$-\frac{\partial \rho}{\partial t} = i_1 - i_2 = i_1 E_1 - i_2 E_2$ (1)

Now

$E_1 d_1 + E_2 d_2 = V$ (2)

$i_1 E_1 = i_2 E_2 = i_2$ (3)

Solving ③ and ④ for E_1 and E_2 in terms of T and L_0 :

$$E_1 = \frac{1}{2} \frac{L_0 T + L_0}{L_0 T + L_0} \quad \text{⑤} \quad E_2 = \frac{1}{2} \frac{L_0 T - L_0}{L_0 T + L_0} \quad \text{⑥}$$

(2) Substituting ⑤ and ⑥ in ②

$$-\frac{L_0}{L_0 T + L_0} = \frac{1}{2} \frac{L_0 T + L_0}{L_0 T + L_0} E_1 + \frac{1}{2} \frac{L_0 T - L_0}{L_0 T + L_0} E_2 \quad \text{⑦}$$

Solution of ⑦:

$$L_0 = \left(\frac{L_0 T + L_0}{L_0 T + L_0} \right) T (1 - e^{-\alpha L_0}) \quad \text{⑧}$$

$$\text{where } T = \text{Relaxation time} = \frac{L_0 T + L_0}{L_0 T + L_0} \quad \text{⑨}$$

(3) Using ⑤ and ⑥:

$$E_1 = \frac{1}{2} \frac{L_0 T + L_0}{L_0 T + L_0} (1 - e^{-\alpha L_0}) = \frac{1}{2} \frac{L_0 T + L_0}{L_0 T + L_0} e^{-\alpha L_0}$$

$$E_2 = \frac{1}{2} \frac{L_0 T - L_0}{L_0 T + L_0} (1 - e^{-\alpha L_0}) = \frac{1}{2} \frac{L_0 T - L_0}{L_0 T + L_0} e^{-\alpha L_0}$$

Ex-11 (2) $E_1 = \frac{1}{2} \frac{L_0 T + L_0}{L_0 T + L_0}$ $E_2 = \frac{1}{2} \frac{L_0 T - L_0}{L_0 T + L_0}$

$$E = E_1 + E_2 = \frac{1}{2} \frac{L_0 T + L_0}{L_0 T + L_0} + \frac{1}{2} \frac{L_0 T - L_0}{L_0 T + L_0}$$

$$E = E_1 + E_2 = \frac{1}{2} \frac{L_0 T + L_0}{L_0 T + L_0} + \frac{1}{2} \frac{L_0 T - L_0}{L_0 T + L_0}$$

(3) $E_1 = \frac{1}{2} \frac{L_0 T + L_0}{L_0 T + L_0} = \frac{1}{2} \frac{L_0 T + L_0}{L_0 T + L_0}$

$$E_2 = \frac{1}{2} \frac{L_0 T - L_0}{L_0 T + L_0} = \frac{1}{2} \frac{L_0 T - L_0}{L_0 T + L_0}$$

$$E_1 = \frac{1}{2} \frac{L_0 T + L_0}{L_0 T + L_0} = \frac{1}{2} \frac{L_0 T + L_0}{L_0 T + L_0}$$

Ex-12 $T = 1 \rightarrow \frac{1}{2} \left(1 - \frac{1}{2} \right) = \frac{1}{4}$

Solution: $V(t) = V_0 \sin \omega t$

Boundary conditions: $V(0) = V_0$; $V(\pi) = 0$

$$\text{--- } V(0) = V_0 \frac{1}{2} \frac{L_0 T + L_0}{L_0 T + L_0}$$

$$V(\pi) = V_0 \frac{1}{2} \frac{L_0 T - L_0}{L_0 T + L_0}$$

$$V(0) = V_0 \frac{1}{2} \frac{L_0 T + L_0}{L_0 T + L_0}$$

$$V = \frac{1}{2} \frac{L_0 T + L_0}{L_0 T + L_0} = \frac{1}{2} \frac{L_0 T + L_0}{L_0 T + L_0} \frac{1}{2} \frac{L_0 T + L_0}{L_0 T + L_0}$$

$$V = \frac{1}{2} \frac{L_0 T + L_0}{L_0 T + L_0} = \frac{1}{2} \frac{L_0 T + L_0}{L_0 T + L_0}$$

Ex. 10 Assume a potential difference V_0 between the inner and outer spheres:

$$\nabla^2 V = 0 \rightarrow \frac{1}{r^2} \frac{d}{dr} (r^2 \frac{dV}{dr}) = 0 \rightarrow V = \frac{A}{r} + B, \quad \frac{dV}{dr} = -\frac{A}{r^2}$$

$$V_0 = -\int_a^b \frac{A}{r^2} dr = -A \left(\frac{1}{a} - \frac{1}{b} \right) \rightarrow A = \frac{bV_0}{\frac{1}{a} - \frac{1}{b}}$$

$$\rightarrow A = \frac{abV_0}{\frac{b-a}{a}} \quad \frac{dV}{dr} = -\frac{bV_0}{b-a} \frac{1}{r^2}$$

$$E = -\frac{dV}{dr} = \frac{bV_0}{b-a} \frac{1}{r^2} \quad \int_a^b E^2 r^2 \sin \theta d\theta dr = \frac{4\pi ab^2 V_0^2}{b-a}$$

$$R = \frac{V_0}{E} = \frac{1}{4\pi ab} \left(\frac{1}{a} - \frac{1}{b} \right), \text{ which can be obtained by combining Eqs. (9-10) and (10-10).}$$

Ex. 11 Assume a current I between the spherical conductors.

$$J = \frac{1}{4\pi r^2} \frac{d\Phi}{dr} = rE,$$

$$V_0 = -\int_a^b E dr = -\int_a^b \frac{1}{r} \frac{d\Phi}{4\pi r} = -\frac{1}{4\pi} \int_a^b \frac{d\Phi}{r^2} \\ = -\frac{1}{4\pi} \int_a^b \frac{1}{r} \left(\frac{1}{a} - \frac{1}{r} \right) dr = -\frac{1}{4\pi} \ln \left(\frac{b(a-1)}{a(b-1)} \right)$$

$$R = \frac{V_0}{I} = \frac{1}{4\pi ab} \ln \left(\frac{b(a-1)}{a(b-1)} \right)$$

Ex. 12 Assume I . $\nabla^2 V = -\frac{I}{r^2} \frac{d\Phi}{dr}$

$$E(r) = \int_a^b \frac{1}{r^2} \frac{d\Phi}{dr} dr = \frac{I}{4\pi r^2} (r - \cos \theta)$$

$$E(r) = \frac{1}{4\pi r^2} \frac{d\Phi}{dr} = \frac{I}{4\pi r^2} \frac{1}{(r - \cos \theta)}$$

$$V_0 = -\int_a^b E(r) dr = -\frac{I}{4\pi} \ln \left(\frac{b(a-1)}{a(b-1)} \right)$$

$$R = \frac{V_0}{I} = \frac{1}{4\pi (a-b)} \left(\frac{1}{a} - \frac{1}{b} \right)$$

Ex. 13 $\nabla \cdot J = 0 = \nabla \cdot (rE) = r \nabla \cdot E + (E \cdot \nabla)r = 0$

$$E = \frac{V_0}{r^2} \hat{r}, \quad \nabla \cdot E = \frac{1}{r^2} \frac{d}{dr} (r^2 E) = \frac{1}{r^2} \frac{d}{dr} (r^2 \frac{V_0}{r^2}) = -\frac{2V_0}{r^3}$$

Defining h such: $R \frac{dR}{dr} = c \rightarrow R = \frac{c}{2} \ln \frac{r}{R_0}$

$$V = -\int_{R_0}^R E dr = c \ln \frac{R}{R_0} \rightarrow c = \frac{1}{2(R_0/R)} E = \frac{1}{2(R_0/R)} \frac{Q}{4\pi R^2 \epsilon_0}$$

$$E = \int_0^R E dr = \int_0^R E dr = \int_0^R \left(\frac{Q}{4\pi R^2 \epsilon_0} \right) \left[\frac{R_0}{R(R_0/R)} \right] dr = \frac{Q}{4\pi R_0 \epsilon_0} \ln \left(\frac{R}{R_0} \right)$$

$$E = \frac{Q}{4\pi R_0 \epsilon_0} \ln \left(\frac{R}{R_0} \right)$$

Ex-22 Assume charges $+q$ and $-q$ to be distributed at the centers of spheres 1 and 2 respectively. Find V_1, V_2



$$V_1 = \frac{q}{4\pi \epsilon_0} \left(\frac{1}{R_1} - \frac{1}{a-R_2} \right)$$

$$V_2 = \frac{q}{4\pi \epsilon_0} \left(\frac{1}{a-R_1} - \frac{1}{R_2} \right)$$

$$C = \frac{q}{V_1 - V_2} = \frac{4\pi \epsilon_0 a}{\frac{1}{R_1} - \frac{1}{R_2} - \frac{1}{a-R_1} + \frac{1}{a-R_2}} = \frac{4\pi \epsilon_0 a}{\frac{1}{R_1} + \frac{1}{R_2} - \frac{2}{a}}$$

$$R = \frac{1}{\frac{1}{4\pi \epsilon_0} \left(\frac{1}{R_1} + \frac{1}{R_2} - \frac{2}{a} \right)} = \frac{1}{\frac{1}{4\pi \epsilon_0} \left(\frac{1}{R_1} + \frac{1}{R_2} - \frac{2}{a} \right)}$$

Ex-23



Plot curved flow pattern of the flow field of Fig. 21. If both the cylinders and its image are fed with the same current, it exactly the same as that of R_0 in all boundary conditions satisfied.

$$V = \left(\frac{q}{4\pi \epsilon_0} \right) = 0 \rightarrow V = 0$$

$$V_0 \text{ can write } V = -E \cdot r$$

where V and electrostatic potential V are simply related. The streamlines are similar to the E -lines of a cylindrical and the image, both carrying a charge q in the electrostatic case.

Ex. 10 According to problem 8.5-10, the current flow pattern would be the same as that of a whole sphere in an undisturbed parallel medium. Hence the current flow would be radial. Assume a current of I :

$$J = J_0 \frac{\vec{r}}{r^2} d\tau, \quad \vec{r} = \vec{r}_0 \frac{r}{r_0},$$

$$J_0 = - \int_V d\tau = - \int_0^{r_0} dr \int_0^\pi d\theta \int_0^{2\pi} d\phi = - \frac{4\pi}{3} r_0^3.$$

$$E = \frac{J_0}{\sigma} = \frac{I}{4\pi\sigma r_0^3} = \frac{I}{4\pi(2.3 \times 10^8)(1)} = 1.1 \times 10^{-4} \text{ (V)}.$$

Ex. 11 Specified boundary conditions can be satisfied by solutions of Laplace's equation with some separation constants: $k_x = k_y = 0$. $X(0) = A_1 x + A_2$, $X(1) = A_3 x + A_4$, $A_1 = A_3 = 0$, $V(x) = A_2 x$.

$$\begin{aligned} \text{a) } \text{At } x=0, \quad V(0) = V_0 = A_2 A_2 x \longrightarrow A_2 A_2 = \frac{V_0}{x} \\ \therefore V = \frac{V_0}{x} x. \end{aligned}$$

$$\text{b) } E = -\partial V = -A_2 \frac{V_0}{x^2} \longrightarrow J = \sigma E = -\sigma A_2 \frac{V_0}{x^2}.$$

Ex. 12 $V(x, y) = \sum_{n=1}^{\infty} (A_n e^{n\pi x} + B_n e^{-n\pi x}) \cos n\pi y$

$$\text{a) } E_1: V(x, 0) = V(x, 1) = 0 \longrightarrow A_n = B_n$$

$$x=0, \quad V_0 = \frac{1}{2} \cos \pi y \longrightarrow A_n \cos n\pi y = \frac{1}{2} \cos \pi y$$

$$\text{Hence } V(x, y) = (A_1 e^{n\pi x} + A_1 e^{-n\pi x}) \cos \pi y, \quad A_1 \cos \pi y = \frac{1}{2} \cos \pi y$$

$$\text{b) } \left. \frac{\partial V}{\partial x} \right|_{x=0} = 0 \longrightarrow A_1 = \frac{1}{2} \cos \pi y, \quad A_1 = \frac{1}{2} \cos \pi y = -\frac{1}{2} \cos \pi y$$

$$\therefore V(x, y) = -\frac{1}{2} (e^{n\pi x} + e^{-n\pi x}) \cos \pi y$$

$$J = -\sigma \vec{\nabla} V = \sigma \left(A_1 \frac{\partial V}{\partial x} + A_2 \frac{\partial V}{\partial y} \right)$$

$$= A_1 A_1 \left(1 - \frac{1}{2} \right) \cos \pi y - A_2 A_2 \left(1 + \frac{1}{2} \right) \sin \pi y$$

$$= A_1 (A_1 \cos \pi y - A_2 \sin \pi y) - \frac{1}{2} (A_1 \cos \pi y + A_2 \sin \pi y)$$

$$= A_1 A_1 - \frac{1}{2} (A_1 \cos \pi y + A_2 \sin \pi y), \quad x=0;$$

$$J = 0, \quad x=1$$

Chapter 4

Static Magnetic Fields

Ex 1



$$\frac{\partial B_x}{\partial y} = \frac{\partial B_y}{\partial x} \quad u_2 = u_1 u_3 \quad \odot$$

$$\frac{\partial B_x}{\partial x} = -\frac{\partial B_y}{\partial y} \quad u_2 = u_1 u_3 \quad \odot$$

$$u_2 = \frac{\partial B_x}{\partial x}$$

Combining $\odot$ and $\odot$

$$\frac{\partial B_x}{\partial x} + u_1^2 u_3 = 0$$

$\rightarrow u_2$ and u_3 are constant

At $x=0$, $u_2 = u_3 = 0$ and $u_1 = u_3$ constant

Substituting u_2 in $\odot$: $u_2 = u_3$ constant. At $x=a$, $u_2 = u_3 = u_1$

$\therefore u_2 = u_3$ constant $\rightarrow y = \frac{\partial B_x}{\partial x}$ constant. From $\odot$

$$u_2 = u_3 \text{ constant} \rightarrow u_1 = \frac{\partial B_x}{\partial x} \text{ constant} \quad \text{From } \odot \text{ and } \odot \rightarrow u_1 = -\frac{\partial B_y}{\partial y} = -\frac{\partial B_x}{\partial x} \text{ (constant)}$$

From $\odot$ and $\odot$: $y = (a - \frac{a}{2})^2 = (\frac{a}{2})^2 \rightarrow B_y$ is a constant

Ex 2 $\frac{\partial B_x}{\partial y} = -\frac{\partial B_y}{\partial x} (B = 0 \text{ at } x=0)$

At $B = B_0$, $B = B_0$

$$\frac{\partial B_x}{\partial y} = 0, \quad \frac{\partial B_x}{\partial x} = -\frac{\partial B_y}{\partial y} u_2, \quad \frac{\partial B_x}{\partial x} = -\frac{\partial B_y}{\partial y} (u_2 - u_1) \quad \begin{cases} u_1 = 0, \\ u_2 = (u_2 - \frac{B_0}{2}) \text{ constant} = \frac{B_0}{2}, \\ u_3 = (\frac{B_0}{2} - u_1) \text{ constant}, \quad u_3 = \frac{B_0}{2} \end{cases}$$

At the interface $x=a$, B_y is constant $\rightarrow$ the magnetic field is constant $B = B_0$, $y = \frac{B_0}{2}$ constant $\rightarrow$ $B = \frac{B_0}{2}$

Ex 3 $B = (1 - \frac{y}{2})^2 = (1 - \frac{y}{2})^2 = \frac{y^2}{4}$

At $\frac{y}{2} = u_2$, $u_2 = u_3$, $u_1 = u_2$

$u_1 = 0$, and $y = u_2$

$$c) E = -\vec{r}_2 E_2 \quad B$$

$$\left. \begin{aligned} \frac{\partial E_2}{\partial t} &= \frac{1}{\mu_0} \vec{r}_2 \nabla \cdot \vec{r}_2 = \vec{r}_2 \cdot \nabla \vec{r}_2 \\ \frac{\partial E_2}{\partial t} &= -\vec{r}_2 E_2 \\ \frac{\partial E_2}{\partial t} &= \frac{1}{\mu_0} E_2 \end{aligned} \right\} \begin{array}{l} \text{Cylindrical} \\ \text{coordinate} \\ \text{system} \end{array} \left. \begin{array}{l} \text{Radial direction perpendicular} \\ \text{to the axis of the cylinder} \end{array} \right\}$$

Ex. 3 Application of Ampere's circuital law.

$$0 < r < a, \quad \vec{B} = \vec{r}_\phi \frac{\partial B_\phi}{\partial r}$$

$$a < r < b, \quad \vec{B} = \vec{r}_\phi \frac{\partial B_\phi}{\partial r}$$

$$r > b, \quad \vec{B} = \vec{r}_\phi \left(\frac{\partial B_\phi}{\partial r} \right) \frac{r}{b}$$

Ex. 4



a) Using $\vec{B}_\phi (0 < r < a)$

$$\vec{B}_\phi = \vec{r}_\phi B_\phi = \vec{r}_\phi \frac{\partial B_\phi}{\partial r}$$

$$= \vec{r}_\phi \frac{\partial}{\partial r} \left(\frac{\mu_0 q}{4\pi r^2} \right) = \vec{r}_\phi \frac{\partial}{\partial r} \left(\frac{\mu_0 q}{4\pi r^2} \right)$$

$$\vec{B}_\phi = -\frac{\mu_0 q}{4\pi r^3} \vec{r}_\phi$$

$$\vec{B}_\phi = -\frac{\mu_0 q}{4\pi r^3} \vec{r}_\phi \quad \text{or } \vec{B}_\phi = -\frac{\mu_0 q}{4\pi r^3} \vec{r}_\phi$$

$$\therefore \vec{B}_\phi = \vec{r}_\phi B_\phi + \vec{r}_\phi \frac{\partial B_\phi}{\partial r}$$

$$\text{where } B_\phi = \frac{\mu_0 q}{4\pi r^3} \int_0^\pi \frac{\partial B_\phi}{\partial r} = \frac{\mu_0 q}{4\pi r^3} \ln \left(\frac{r}{a} \right)$$

$$B_\phi = \frac{\mu_0 q}{4\pi r^3} \int_0^\pi \frac{\partial B_\phi}{\partial r} = \frac{\mu_0 q}{4\pi r^3} \ln \left(\frac{r}{a} \right)$$

b) To find $\vec{B}$ at $\vec{r}$ (Fig. 1), we add vectorially the contributions of the current loops to the right and to the left of point $\vec{r}$ using the result in part (a).

$$\vec{B}_\phi = \vec{B}_\phi + \vec{B}_\phi$$

$$\vec{B}_\phi = \frac{\mu_0 q}{4\pi r^3} \left[\vec{r}_\phi \ln \left(\frac{r}{a} \right) + \vec{r}_\phi \ln \left(\frac{r}{b} \right) \right]$$

$$\vec{B}_\phi = \frac{\mu_0 q}{4\pi r^3} \left[\vec{r}_\phi \ln \left(\frac{r}{a} \right) + \vec{r}_\phi \ln \left(\frac{r}{b} \right) \right]$$

$$\therefore \vec{B}_\phi = \frac{\mu_0 q}{4\pi r^3} \left[\vec{r}_\phi \left(\ln \left(\frac{r}{a} \right) + \ln \left(\frac{r}{b} \right) \right) + \vec{r}_\phi \ln \left(\frac{r}{\sqrt{a^2 + b^2}} \right) \right]$$

Ex. 1 From Example 4-6, Eq. (4-14),



The value of $\vec{H}$ is determined by the right-hand rule

$$\begin{aligned} d\vec{H} &= \frac{\mu_0 I b dx}{2\pi r^3} \left(\frac{y}{r} \right) \hat{e}_1 \\ &= \frac{\mu_0 I b dx}{2\pi} \int_0^l \frac{y}{(x^2 + y^2 + z^2)^{3/2}} \\ &= \frac{\mu_0 I b}{2\pi} \left[\frac{xy}{(x^2 + y^2 + z^2)^{3/2}} \right. \\ &\quad \left. - \frac{y}{2(x^2 + y^2 + z^2)^{1/2}} \right]_{x=0}^{x=l} \end{aligned}$$

$$\vec{H} = H_x \hat{e}_1$$

At $P(x, y)$ the magnitude of the density due to an infinitely long strip of width $b dx$ is

$$\begin{aligned} dH &= \frac{\mu_0 I b dx}{2\pi} \left(\frac{y}{r} + \frac{x^2}{r^3} \right) \\ r &= \sqrt{x^2 + y^2 + z^2} \end{aligned}$$

$$\therefore \vec{H} = \int d\vec{H} = H_x \hat{e}_1 + H_y \hat{e}_2$$

where

$$\begin{aligned} H_x &= \frac{\mu_0 I b}{2\pi} \int_{-a}^a \frac{xy}{(x^2 + y^2 + z^2)^{3/2}} \\ &= \frac{\mu_0 I b}{2\pi} \left[\ln \frac{(x^2 + y^2 + z^2)^{1/2} + x}{(x^2 + y^2 + z^2)^{1/2} - x} \right] \end{aligned}$$

$$\begin{aligned} H_y &= \frac{\mu_0 I b}{2\pi} \int_{-a}^a \frac{x^2}{(x^2 + y^2 + z^2)^{3/2}} \\ &= \frac{\mu_0 I b}{2\pi} \ln \frac{(x^2 + y^2 + z^2)^{1/2} + x}{(x^2 + y^2 + z^2)^{1/2} - x} \end{aligned}$$



Top view

Ex. 2 This problem is a superposition of two problems:

$$\vec{H} = \vec{H}_1 + \vec{H}_2$$

where

1. $\vec{H}_1$ is the magnetic flux density at P due to the

Simultaneously sending out equal and opposite currents. Assuming R_p positive and ℓ as before:

$$E_1 = E_0 \frac{R_p \ell}{2a^2}$$

2. E_2 is the magnitude of the electric field due to a right circle. Taking one half of the circle as $E_p(1-a)$ the result:

$$E_2 = E_0 \frac{R_p \ell}{2a^2}$$

$$\therefore E = E_0 \frac{R_p \ell}{2a^2} \left(\frac{1}{a} + \frac{1}{a} \right)$$

Ex-22 Use $E_p(1-a)$ $E_2 = \frac{R_p \ell}{2a^2} \left\{ \frac{1}{\sqrt{1-a^2/a^2}} - \frac{1}{\sqrt{1-a^2/a^2}} \right\}$.



$$\text{a) At } a=0, E_2 = \frac{R_p \ell}{2a^2} \frac{1}{\sqrt{1-a^2/a^2}}$$

$$\text{so } \frac{dE_2}{da} = \frac{R_p \ell}{2a^2} \left\{ -\frac{1}{\sqrt{1-a^2/a^2}} \cdot \frac{2a}{1-a^2/a^2} + \frac{1}{\sqrt{1-a^2/a^2}} \right\}$$

At the endpoint, $a=0$, $\frac{dE_2}{da} = 0$.

$$\text{c) } \frac{dE_2}{da} = -\frac{R_p \ell}{2a^2} \left\{ \frac{1}{\sqrt{1-a^2/a^2}} - \frac{1}{\sqrt{1-a^2/a^2}} \cdot \frac{2a}{1-a^2/a^2} \right\}$$

$$\text{At } a=0, \frac{dE_2}{da} = -\frac{R_p \ell}{2a^2} \left\{ \frac{1}{\sqrt{1-a^2/a^2}} \right\} \rightarrow 0 \text{ as } a \rightarrow 0$$

Ex-23 Use $E_p(1-a)$ for a wire of length $2a$.

$$E = E_0 \frac{R_p \ell}{2a^2} \frac{1}{\sqrt{1-a^2/a^2}}$$



In this problem, $a = \frac{R_p \ell}{2a^2}$, $\frac{dE}{da} = \frac{R_p \ell}{2a^2} \frac{1}{\sqrt{1-a^2/a^2}}$

$$E = E_0 \frac{R_p \ell}{2a^2} \left(\frac{1}{\sqrt{1-a^2/a^2}} \right) = E_0 \frac{R_p \ell}{2a^2} \frac{1}{\sqrt{1-a^2/a^2}}$$

When a is very large, $\frac{R_p \ell}{2a^2} \approx \frac{R_p \ell}{2a^2}$, $E \rightarrow E_0 \frac{R_p \ell}{2a^2}$, which is the same as $E_p(1-a)$ with $a=0$.

Ex. 11 $E_z = \frac{qV_0}{4\pi\epsilon_0 r^2}$, $E = \int E_z dz = \frac{qV_0}{4\pi\epsilon_0} \int_{-\infty}^{\infty} \frac{1}{r^2} dz = \frac{qV_0}{4\pi\epsilon_0} \frac{2}{r}$

If E_z at $r = \frac{R}{2}$ is used, $E = \frac{qV_0}{4\pi\epsilon_0} \left(\frac{4}{R} \right)$

to agree $= \frac{qV_0}{2\pi\epsilon_0 R} \text{ correct} = \left[\frac{4qV_0}{4\pi\epsilon_0 R} - 1 \right] \frac{qV_0}{2\pi\epsilon_0 R}$

Ex. 12 $E = E_1 + E_2$, $\oint E \cdot d\mathbf{l} = \rho_{\text{enc}} l$



If there is no hole,

$$\sigma_1 R_1 = \rho_{\text{enc}} r$$

$$\rightarrow E_1 = \frac{\rho_{\text{enc}} r}{2\epsilon_0} \Rightarrow \begin{cases} E_1 = \frac{\sigma_1 R_1}{2\epsilon_0} \\ E_2 = \frac{\sigma_2 R_2}{2\epsilon_0} \end{cases}$$

For $r = R_1$ in the hole portion,

$$E_1 = \frac{\rho_{\text{enc}} r}{2\epsilon_0} \Rightarrow \begin{cases} E_1 = \frac{\sigma_1 R_1}{2\epsilon_0} \\ E_2 = \frac{\sigma_2 R_2}{2\epsilon_0} \end{cases}$$

Superposing E_1 and E_2 and noting that $\sigma_1 R_1$ and $\sigma_2 R_2$ are

we have $E_1 = E_{11} + E_{12} = 0$, and $E_2 = E_{21} + E_{22} = \frac{\sigma_2 R_2}{2\epsilon_0}$

Ex. 13 $E = 0 = E_1 \Rightarrow E = E_1 + E_2 = E_1 \left(\frac{R_1}{r} - \frac{R_2}{r} \right) = -E_1 \frac{R_2}{r}$

For $0 < r < R_1$, $d\rho(r)dl$ gives $E = E_1 \frac{R_1}{r}$

For $r > R_2$, $d\rho(r)dl$ gives $E = E_2 \frac{R_2}{r}$

Integrating, $E_1 = E_1 \left[\frac{R_1}{r} \left(\frac{1}{R_1} - \frac{1}{R_2} \right) + C_1 \right]$, $0 < r < R_1$

$$E_2 = E_2 \left[\frac{R_2}{r} \left(\frac{1}{R_2} - \frac{1}{R_1} \right) + C_2 \right]$$
, $r > R_2$

At $r = R_1$, $E_1 = E_2 \Rightarrow C_1 = \frac{R_2}{R_1} \left(\frac{1}{R_2} - \frac{1}{R_1} \right) + C_2$

$$\therefore E_1 = E_2 \left[-\frac{R_2}{R_1} \left(\frac{1}{R_2} - \frac{1}{R_1} \right) + C_2 \right]$$
, $r > R_2$

Ex. 14 E_p (Ex. 13) for one wire: $E = E_1 \frac{R_1}{r} \ln \left(\frac{R_2 + R_1}{R_2 - R_1} \right)$

For two wires carrying equal and opposite currents,

a) $E = E_1 \frac{R_1}{r} \ln \left[\frac{R_2 + R_1}{R_2 - R_1} + \frac{R_2 + R_1}{R_2 - R_1} \right] = E_1 \frac{R_1}{r} \ln \left[\frac{2}{1} \frac{R_2 + R_1}{R_2 - R_1} \right]$

b) For a very long horizontal transmission line, assume

$$R = R_0 \frac{y^2}{x^2} \ln\left(\frac{x}{y}\right) = R_0 \frac{y^2}{x^2} \ln\left(\frac{\frac{1}{2} + \sqrt{\frac{1}{4} + y^2}}{\frac{1}{2} - \sqrt{\frac{1}{4} + y^2}}\right).$$

$$c) R = \phi_0 R = R_0 \frac{y^2}{x^2} = R_0 \frac{y^2}{x^2}$$

$$= R_0 \frac{y^2}{x^2} \left[\frac{1}{\sqrt{1+y^2}} - \frac{1}{\sqrt{1+y^2}} \right] = R_0 \frac{y^2}{x^2} \left[\frac{1}{\sqrt{1+y^2}} - \frac{1}{\sqrt{1+y^2}} \right]$$

$$= R_0 \left[\frac{1}{2} - \frac{1}{2} \right]$$

or To find the equation for magnetic flux lines

$$\frac{dy}{dx} = \frac{R}{R_0} \implies \frac{dy}{dx} = \frac{y^2}{x^2} \implies \frac{dy}{y^2} = \frac{dx}{x^2} \implies -\frac{1}{y} = -\frac{1}{x} + C$$

$$\text{Then } \frac{dy}{dx} = \frac{1}{x} \implies y = \ln x + C$$

Ex-12 Apply divergence theorem to (P, Q, R) where C is a constant vector.

$$\int_V \nabla \cdot (P, Q, R) dV = \int_S (P, Q, R) \cdot d\mathbf{S} \quad (1)$$

Now, from problem 12-11: $\nabla \cdot (P, Q, R) = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z}$

$$= \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z} \quad (2)$$

$$\text{From Ex-11-41: } \nabla \cdot (P, Q, R) = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z} \quad (3)$$

Substituting (2) and (3) in (1)

$$\int_V \left(\frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z} \right) dV = \int_S (P, Q, R) \cdot d\mathbf{S}$$

Ex-13

$$\int_V \nabla \cdot \mathbf{F} dV$$

$$a) \text{ Given } \mathbf{F} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$$

$$\mathbf{F} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k} \implies \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z} = 1 + 1 + 1 = 3$$

$$b) \text{ Given } \mathbf{F} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$$

$$\mathbf{F} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k} \implies \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z} = 1 + 1 + 1 = 3$$

Ex-14 a) Given

$$\mathbf{F} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$$

$$\text{b) Given } \mathbf{F} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$$

$$\mathbf{F} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$$

$$\mathbf{F} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$$

$$c) \text{ Given } \mathbf{F} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k} \implies \mathbf{F} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k} \implies \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z} = 1 + 1 + 1 = 3$$

Ex. 14.12



so $\vec{B} = \mu_0 I \oint \frac{d\vec{l} \times \vec{r}}{r^3} = -\vec{a}_z \mu_0 \frac{IR^2}{2} \int_0^{2\pi} \frac{1}{(R^2+z^2)^{3/2}} dz$, which is the same as $\frac{\mu_0 I R^2}{2(R^2+z^2)^{3/2}}$.

Ex. 14.13 A cylindrical bar magnet having a uniform magnetic induction $B_0 \vec{a}_z$ is equivalent to $\vec{m} = \vec{a}_z \pi R^2 \Delta z$ and so $\vec{B}_{\text{eq}} = \frac{\mu_0}{4\pi} \frac{\vec{m} \times \vec{r}}{r^3} = \frac{\mu_0}{4\pi} \frac{\vec{a}_z \pi R^2 \Delta z \times \vec{r}}{r^3}$ on the cylinder wall. At a distant point, $\vec{B}$ due to this $\vec{m}$ following on a cylindrical shell of length Δz and radius R is the same as that due to a circular loop of radius R carrying a current $I = \frac{B_0 R}{\mu_0}$. It is given by Eq. (14-41), which is the same as Eq. (14-42) obtained in Example 14-12 where the total dipole moment of the cylindrical magnet is $\vec{m} = \vec{a}_z \pi R^2 \Delta z$.

Ex. 14.14



a) $\vec{B}_0 = B_0 \vec{a}_z = 0$

$$\vec{B}_{\text{eq}} = \frac{\mu_0}{4\pi} \oint \frac{d\vec{l} \times \vec{r}}{r^3} = \vec{a}_z \oint \frac{dl}{r^2} = \vec{a}_z \oint \frac{dl}{R^2}$$

b) Apply Eq. (14-41) to a loop of radius R that carries a current I_{eq} shell.

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{d\vec{l} \times \vec{r}}{r^3} = \frac{\mu_0}{4\pi} \frac{dl \sin \theta}{R^2} \vec{a}_z$$

$$\vec{B} = \oint d\vec{B} = \vec{a}_z \frac{\mu_0 I_{\text{eq}}}{4\pi R^2} \int_0^{2\pi} dl \sin \theta = \vec{a}_z \frac{1}{2} \mu_0 I_{\text{eq}} = \frac{1}{2} \mu_0 B_0$$



$$\vec{E}_1 = \frac{\partial \vec{A}_1}{\partial t} = (-E_1 \sin k_1 z + E_1 \sin k_1 z) \hat{y}$$

$$\vec{E}_2 = \frac{\partial \vec{A}_2}{\partial t} = (-E_2 \sin k_2 z + E_2 \sin k_2 z) \hat{y}$$

$$\therefore \vec{E} = \vec{E}_1 + \vec{E}_2$$

$$= -E_1 \frac{\partial}{\partial t} \left[\frac{1}{\sqrt{1-k_1^2}} \cos k_1 z + \frac{1}{\sqrt{1-k_2^2}} \cos k_2 z \right] \\ = E_1 \frac{\partial}{\partial t} \left[\frac{1}{\sqrt{1-k_1^2}} \sin k_1 z + \frac{1}{\sqrt{1-k_2^2}} \sin k_2 z \right]$$



$$\text{a) (i) If } z = \text{max, } \vec{E}_1 = \vec{E}_2 = 0$$

$$B_0 \text{ continuous} \rightarrow B_{\text{ext}} = B_0$$

$$E_{\text{ext}} = E_1 = E_2 \rightarrow E = -E_1 B_0$$

$$\text{Image } E_1 \text{ is } \frac{1}{\sqrt{1-k_1^2}} \sin k_1 z \text{ at } z = \text{max}$$

$$\text{If } z = \text{max, } \vec{E}_1 = 0, \text{ but } \vec{E}_2 \text{ is finite}$$

$$\text{No surface current} \rightarrow B_{\text{ext}} = B_0$$

$$B_0 \text{ continuous} \rightarrow B_{\text{ext}} = B_0$$

$$\text{Image } E_2 \text{ is } \frac{1}{\sqrt{1-k_2^2}} \sin k_2 z \text{ at } z = \text{max}$$

$$\text{b) } \vec{E}_1 = \vec{E}_2 = (E_1)_z \hat{z} = E_1 \sin k_1 z \hat{z}$$

$$\text{c) } \vec{E}_2 = \vec{E}_1 = (E_2)_z \hat{z} = E_2 \sin k_2 z \hat{z}$$

$$\text{d) } \vec{E}_1 = \vec{E}_2 = (E_1)_z \hat{z} = E_1 \sin k_1 z \hat{z}$$

$$\text{e) } \vec{E}_1 = -E_1 \sin k_1 z \hat{z} = E_1 \left(\frac{1}{\sqrt{1-k_1^2}} \right) \hat{z}$$

$$\text{f) } \vec{E}_2 = 0$$

8.16



$$\vec{E} = \vec{E}_1 + \vec{E}_2 = E_1 \sin k_1 z \hat{z} + E_2 \sin k_2 z \hat{z}$$

$$\vec{E} = \frac{\partial \vec{A}}{\partial t} = \frac{\partial}{\partial t} \left[\frac{1}{\sqrt{1-k_1^2}} \cos k_1 z + \frac{1}{\sqrt{1-k_2^2}} \cos k_2 z \right] \hat{z}$$

$$\therefore E = \frac{\partial}{\partial t} \left[\frac{1}{\sqrt{1-k_1^2}} \cos k_1 z + \frac{1}{\sqrt{1-k_2^2}} \cos k_2 z \right]$$

$$\text{If } z = \text{max, } E_1 = \frac{\partial}{\partial t} \left[\frac{1}{\sqrt{1-k_1^2}} \cos k_1 z \right]$$

$$E = E_1 = E_2 \text{ at } z = \text{max} = \frac{\partial}{\partial t} \left[\frac{1}{\sqrt{1-k_1^2}} \cos k_1 z + \frac{1}{\sqrt{1-k_2^2}} \cos k_2 z \right]$$

Ex. 26 For $b \rightarrow a$ ($b \rightarrow 0$), $E_2 = 2E_1 - 2q \frac{\rho_0}{\epsilon_0} \left[1 - \frac{\sqrt{1 - \frac{a^2}{b^2}}}{\sqrt{1 - \frac{a^2}{b^2} - \frac{a^2}{b^2}}} \right]$
 $= 2q \frac{\rho_0}{\epsilon_0} \left[\frac{2\sqrt{1 - \frac{a^2}{b^2}}}{\sqrt{1 - \frac{a^2}{b^2} - \frac{a^2}{b^2}}} \right]$.

Magnetic energy per unit length stored in the other conductors

$$W'_2 = \frac{1}{2} \mu_0 \int_0^{b-a} E'_2 z \, dz$$

$$= \frac{\mu_0}{4\pi} \left\{ \frac{(\frac{2q\rho_0}{\epsilon_0})^2}{\sqrt{1 - \frac{a^2}{b^2} - \frac{a^2}{b^2}}} \ln(1 + \frac{1}{\sqrt{1 - \frac{a^2}{b^2} - \frac{a^2}{b^2}}}) \right\}$$

From Eqs. (26-10), (26-11), and (26-12) we have,

$$C' = \frac{1}{2} (W'_1 + W'_2 + W'_3)$$

$$= \frac{\rho_0}{2\epsilon_0} \left[\frac{1}{2} + \ln \frac{1}{2} + \frac{(\frac{2q\rho_0}{\epsilon_0})^2}{\sqrt{1 - \frac{a^2}{b^2} - \frac{a^2}{b^2}}} \ln(1 + \frac{1}{\sqrt{1 - \frac{a^2}{b^2} - \frac{a^2}{b^2}}}) \right] \ln$$

Ex. 27



E at a distance r from one infinitely long line carrying a current I : $E = 2q \frac{\rho_0}{\epsilon_0}$

E' is unit length wire above due to I by Amp. & that due to the other line from B to E' is

$$E_2 = \frac{q\rho_0}{\epsilon_0} \int_{2a}^{r+a} \frac{dz}{z^2} = \frac{q\rho_0}{\epsilon_0} \ln \frac{r}{2a}$$

That due to E is E_1 is

$$E_1 = \frac{q\rho_0}{\epsilon_0} \ln \frac{r}{2a}$$

Total flux linkage per unit length

$$L_{12} = E_1 - E_2 = \frac{q\rho_0}{\epsilon_0} \ln \frac{(\frac{r}{2a})^2}{\frac{r}{2a}}$$

$$= \frac{q\rho_0}{\epsilon_0} \ln \frac{r}{2a} - \frac{q\rho_0}{\epsilon_0} \ln \frac{r}{2a}$$

$$\therefore L_{12} = \frac{q\rho_0}{\epsilon_0} = \frac{q\rho_0}{\epsilon_0} \ln(1 + \frac{1}{2})$$

Ex. 28 For I in the long straight wire, $E = 2q \frac{\rho_0}{\epsilon_0}$

$$L_{12} = \int E \, dz - \int E_2 \, dz = \frac{q\rho_0}{\epsilon_0} \int_{2a}^{r+a} \frac{dz}{z^2} \ln$$

$$= \frac{q\rho_0}{\epsilon_0} \left[\ln \ln \ln(1 + \frac{1}{2}) \right] \rightarrow L_{12} = \frac{q\rho_0}{\epsilon_0} \left[\frac{1}{2} - \frac{1}{2} \ln(1 + \frac{1}{2}) \right]$$

Ex-40



Assume a current I .

$$B \text{ at } P \text{ due to } B_0 = \frac{\mu_0 I}{2\pi(a+R \cos \theta)}$$

$$\begin{aligned} \vec{A}_{\text{net}} &= \frac{\mu_0 I}{2\pi} \int_0^{2\pi} \frac{R \sin \theta d\theta}{a+R \cos \theta} \\ &= \frac{\mu_0 I}{2\pi} \int_0^{2\pi} \frac{R \sin \theta d\theta}{a+R \cos \theta} = \mu_0 I R \int_0^{2\pi} \frac{\sin \theta d\theta}{a+R \cos \theta} \\ \vec{A}_{\text{net}} &= \mu_0 I R \left(\sqrt{a^2 - R^2} \right) \end{aligned}$$

Ex-41 Approximate the magnetic flux due to the long loop linking with the small loop by that due to that infinitely long wires carrying equal and opposite current I .

$$\begin{aligned} \vec{A}_{\text{net}} &= \frac{\mu_0 I R}{2\pi} \int_0^{2\pi} \left(\frac{1}{a+R \cos \theta} - \frac{1}{a-R \cos \theta} \right) d\theta = \frac{\mu_0 I R}{2\pi} \ln \left(\frac{a+R}{a-R} \right) \\ \vec{A}_{\text{net}} &= \frac{\mu_0 I R}{2} = \frac{\mu_0 I R}{2\pi} \ln \left(\frac{a+R \cos \theta}{a-R \cos \theta} \right) \end{aligned}$$

Ex-42 $\vec{B}_0 = \frac{\mu_0}{4\pi} \left(\vec{I}_1 \times \frac{\vec{r}_1}{r_1^3} + \vec{I}_2 \times \frac{\vec{r}_2}{r_2^3} + \vec{I}_3 \times \frac{\vec{r}_3}{r_3^3} \right)$

$$\begin{aligned} a) \vec{B}_0 &= \frac{\mu_0}{4\pi} \left[\vec{I}_1 \times \left(\frac{\vec{r}_1}{r_1^3} \right) + \vec{I}_2 \times \left(\frac{\vec{r}_2}{r_2^3} \right) + \vec{I}_3 \times \left(\frac{\vec{r}_3}{r_3^3} \right) \right] = \frac{\mu_0}{4\pi} \left[\vec{I}_1 \times \left(\frac{\vec{r}_1}{r_1^3} + \frac{\vec{r}_2}{r_2^3} + \frac{\vec{r}_3}{r_3^3} \right) \right] \\ \frac{\partial \vec{B}_0}{\partial x} &= \frac{\mu_0}{4\pi} \left(2\vec{I}_1 + 2\vec{I}_2 \right) = 0, \quad \frac{\partial \vec{B}_0}{\partial y} = 2\vec{I}_3 \times \vec{r}_3 > 0 \\ \therefore a &= \frac{\vec{r}_3}{r_3} = -\frac{y}{x} \text{ for minimum } \vec{B}_0 \end{aligned}$$

$$b) \vec{B}_0 \cdot \vec{A}_{\text{net}} = \frac{\mu_0}{4\pi} \left(-\frac{y}{x} + \vec{r}_3 \right) \cdot \vec{r}_3 = \mu_0 \vec{r}_3 \cdot \vec{r}_3$$

Ex-43



$$\vec{I}_1 = \vec{I}_2 = \vec{I}_3 = 2I \cos 30^\circ, \quad a = 2 \cos 30^\circ$$

$$\vec{B}_0 = \vec{B}_1 + \vec{B}_2 + \vec{B}_3 = \vec{I}_1 \times \frac{\vec{r}_1}{r_1^3}$$

Force per unit length on wire 1:

$$\begin{aligned} \vec{F}_1 &= -\vec{I}_1 \times \vec{B}_2 + \vec{I}_1 \times \vec{B}_3 \\ &= -\vec{I}_1 \times \vec{I}_2 \times \frac{\vec{r}_2}{r_2^3} + \vec{I}_1 \times \vec{I}_3 \times \frac{\vec{r}_3}{r_3^3} \end{aligned}$$

Forces on all three wires are equal in magnitude and towards the center of the triangle.

Ex-41 Magnetic field intensity at the wire due to the current of $I = \frac{q}{t} dy$ in an elemental strip is



$$dH = \frac{\mu_0 I}{4\pi r^2} = \frac{\mu_0 I dy}{4\pi r^2 \sqrt{a^2 + y^2}}$$

Symmetry $\rightarrow$ H at the wire has only y -component.

$$H = \mu_0 I \int \left(\frac{y}{r^3} \right) dy = \mu_0 I \int \frac{y dy}{a^2 \sqrt{1 + \frac{y^2}{a^2}}} \\ = \mu_0 \frac{I}{a} \tan^{-1} \left(\frac{y}{a} \right)$$

$$F = H \times dl = (-\mu_0 I) \times (\mu_0 I) = \mu_0 I^2 \frac{dy}{a} \tan^{-1} \left(\frac{y}{a} \right) \quad (14.4)$$

Ex-42 From Problem Ex-41 we have the y -component of the magnetic force density at an arbitrary point P on the right-hand strip due to I_1 in the left-hand strip

$$dF_{1y} = - \frac{\mu_0 I_1^2}{4\pi a^2} \left[\tan^{-1} \left(\frac{y}{a} \right) - \tan^{-1} \left(\frac{y+b}{a} \right) \right] dy$$

The y -component of the force on a strip of width dy due to I_2 in the right-hand conductor is

$$dF_{2y} = \left(\frac{\mu_0 I_2}{2a} dy \right) dF_{1y} \quad (\text{in the } +y \text{ direction, a repulsive force})$$

$$F_y = \mu_0 I_1 I_2 \int \frac{dy}{4\pi a^2} \int_0^b \left[\tan^{-1} \left(\frac{y}{a} \right) - \tan^{-1} \left(\frac{y+b}{a} \right) \right] dy \\ = \mu_0 I_1 I_2 \frac{b^2}{4\pi a^2} \left[1 - \tan^{-1} \left(\frac{b}{a} \right) - \tan^{-1} \left(1 + \frac{b}{a} \right) \right] \quad \text{for } a \ll b \text{ and } b \ll a$$

There is no net force in the y -direction.

Ex-43 H due to I_1 in the straight wire in the x -direction at an elemental wire dl in the circular loop is



F has no net y -component

$$dH = \frac{\mu_0 I_1}{4\pi r^2} \sin \theta$$

$$F_x = -\mu_0 I_1 I_2 \int_0^{2\pi} \left(\frac{I_2 b \sin \theta}{4\pi} \right) \cos \theta \\ = -\mu_0 I_1 I_2 \frac{b^2}{4\pi} \int_0^{2\pi} \frac{\sin^2 \theta \cos \theta}{r^2} d\theta \\ = \mu_0 I_1 I_2 \frac{b^2}{4\pi} \left[\frac{\sin^3 \theta}{3} + C \right] \quad (\text{Applied Book})$$

Ex. 4-16 $\vec{B} = B_0 \frac{dy}{dx} \left(\frac{y}{x} + x \frac{dy}{dx} \right)$, $d\vec{L} = B_0 dy \hat{y}$
 $d\vec{\tau} = \vec{r} \times d\vec{L} = B$
 $= -B_0 \frac{dy}{dx} \left(\frac{y}{x} + x \frac{dy}{dx} \right) dy$
 $\vec{\tau} = -B_0 \frac{dy}{dx} \left(\frac{y^2}{2x} + \frac{1}{2} y^2 \right)$
 $= -B_0 \frac{dy^2}{dx} \ln \left(\frac{y}{x} - \frac{1}{2} \right)$

(A tricky problem.)

Ex. 4-17

Diagram: A triangular loop with side length a is placed in a uniform magnetic field B directed vertically upwards. The forces on the sides are shown: the bottom side has a downward force, and the two slanted sides have upward forces at their top vertices.

Distances $d = \frac{1}{2}a$, $\frac{1}{2}a$, $\frac{1}{2}a$
 $(B_0 \sin 60^\circ \frac{1}{2}a - B_0 \sin 60^\circ \frac{1}{2}a) \sin 60^\circ a$
 Sum $B_0 a \sin 60^\circ$, $B_0 a \sin 60^\circ$, $B_0 a \sin 60^\circ$
 Direction: up!
 $\vec{\tau} = \vec{r} \times \vec{F} = (B_0 \frac{1}{2}a) \times \vec{F}$
 $= (B_0 \frac{1}{2}a \sin 60^\circ) \sin 60^\circ a = B_0 a^2 \sin^2 60^\circ$

Ex. 4-18 Let μ -wire be the center line of the circular cable.
 The magnetic energy stored in a section of length x is
 $U_L = \frac{1}{2} L I^2$

$$L = \frac{\mu}{2} = \frac{\mu}{2} \int_0^x B_z dx = \frac{\mu}{2} \int_0^x \frac{B_0}{\sqrt{1+x^2}} dx = \frac{\mu B_0}{2} \ln \frac{x}{1}$$

$$F_z = B_0 \frac{dU_L}{dx} = B_0 \left(\frac{\mu B_0}{2} \right) \frac{1}{x} = B_0 \frac{dU_L}{dx} \ln \frac{x}{1}$$

Ex. 4-19 Resolve the circular loop into many small loops, each with a magnetic dipole moment $d\vec{\mu} = I_0 d\vec{L}$, $d\vec{\tau} = d\vec{\mu} \times \vec{B}$
 $\vec{\tau} = \int d\vec{\tau} = I_0 \int d\vec{L} \times \vec{B} = I_0 \int d\vec{L} \times B_0 \hat{z} = -B_0 I_0 \int d\vec{L} \times \hat{z}$
 From Problem 4-16

— This torque is in the direction of aligning the flux produced by I_0 in the loop with that of $\vec{B}$ due to I_0 in the straight wire.

Ex-11 $\vec{E}_0$ at the of the large spherical part of wire carrying a current I_0 is (by parting out in Ex 1-11):

$$\vec{E}_0 = E_0 \frac{\vec{r}}{r^2}$$

Then, on the small circular wire, $\vec{r} = \vec{r}_0 + \vec{r}_1$ so $(\vec{r}_0 + \vec{r}_1)^2 = (\vec{r}_0^2 + \vec{r}_1^2 + 2\vec{r}_0 \cdot \vec{r}_1)$
~~is independent of θ since $\vec{r}_0$ is a direction~~
 to align the magnetic flux produced by I_0 & I_1 .

Ex-12



$$\begin{aligned} E_0 &= \text{imagined dipole inside} \\ &= \frac{1}{4\pi\epsilon_0} (q_0 \cos \theta + q_1 \sin \theta) \frac{\vec{r}}{r^2} \\ &= \frac{1}{4\pi\epsilon_0} \left(\frac{q_0}{R} \cos \theta + \frac{q_1}{R} \sin \theta \right) \frac{\vec{r}}{r^2} \\ &= \frac{1}{4\pi\epsilon_0} \frac{1}{R^2} (q_0 \cos \theta + q_1 \sin \theta) \frac{\vec{r}}{r^2} \\ E_0(\cos \theta) &= \frac{1}{4\pi\epsilon_0} \frac{q_0}{R^2} \frac{1}{r^2} \\ \text{Max. deflection occurs when } \left| \frac{\partial}{\partial \theta} \right| & \text{ is max. or when} \\ \left| \frac{\partial}{\partial \theta} \right| &= \left| \frac{d}{d\theta} \left(\frac{1}{r^2} \cos \theta \right) \right| \text{ is min.} \end{aligned}$$

$$\text{Set } \frac{d}{d\theta} \left(\frac{1}{r^2} \cos \theta \right) = 0 \implies \sin \theta = 2z/R, \text{ or } \theta = 2\theta^*$$

$$\text{At } \theta = 2\theta^*, |E_0/E_0| = 1.47, \text{ and at other } \theta, 1.47 = R/z^2$$

$$\text{Ex-13 } F = \frac{q^2}{4\pi\epsilon_0} = \frac{(1.6 \times 10^{-19})^2}{4\pi(8.85 \times 10^{-12})} = \frac{(1.6 \times 10^{-19})^2}{(4\pi \times 10^{-12})}$$

$$F = 1.6 \times 10^{-19} \times 1.47 \times 10^{-12} = 2.35 \times 10^{-31} \text{ N}$$

$$I_1 = 1.47 \mu\text{A}$$

$$\text{Radius: } \text{area} = \pi r^2 = 0.13 \times 10^{-6} \text{ m}^2$$

$$\text{Ex-14 } W_L = \frac{1}{2} \int \rho(r) dr$$

Assume a virtual displacement δu of the free end.

$$W_L(u + \delta u) = W_L(u) + \frac{1}{2} \int_{\text{axis}} (\rho(r) \delta u) dr$$

$$= W_L(u) + \frac{1}{2} \rho_0 (\rho_0 - G \theta^2 I^2) \delta u$$

$$(F_L)_u = \frac{\partial W_L}{\partial u} = \frac{1}{2} \rho_0 (G \theta^2 I^2) \delta u \text{ is the variation of } W_L$$

Chapter 7

Time-Varying Field and Maxwell's Equations

Ex.1 $\nabla \cdot \mathbf{E} = \oint_V \frac{\partial \rho}{\partial t} dV = -\oint_V \frac{\partial}{\partial t} (\rho_0 e^{-t} \cos \theta) dV = -\frac{\partial}{\partial t} \oint_V \rho_0 e^{-t} dV$

Ex.2 $\mathbf{E} = E_0 \sin(\omega t) \cos(\theta) \hat{r} = f(\omega t) \cos(\theta) \hat{r}$

$$\oint_V \mathbf{E} \cdot d\mathbf{r} = \int_0^{2\pi} \int_0^\pi \int_0^R E_0 \sin(\omega t) \cos(\theta) (r^2 \sin \theta dr d\theta d\phi) \quad \omega t = \text{const.}$$

$$= -\frac{E_0}{3} \frac{\partial}{\partial t} [\sin(\omega t) \cos(\theta) - \sin(\omega t) \cos(\pi)] \int_0^{2\pi} d\phi \int_0^\pi \sin \theta d\theta$$

$$\nabla \cdot \mathbf{E} = -\frac{E_0}{3} \int_V \frac{\partial}{\partial t} \mathbf{E} \cdot d\mathbf{r} = \frac{E_0}{3} [\sin(\omega t) \cos(\theta) - \sin(\omega t) \cos(\pi)] \int_V dV$$

$$\rho = \frac{\partial \rho}{\partial t} = \frac{1}{3} E_0 [\sin(\omega t) \cos(\theta) - \sin(\omega t) \cos(\pi)]$$

$$= \frac{1}{3} E_0 \sin(\omega t) (\cos \theta + 1) \quad \text{--- (a)}$$

Ex.3 In the rectangular loop with the assigned direction for $\hat{L}$

$$L_{\text{enc}} \frac{dE_0}{dt} = L \frac{dE_0}{dt} + R L_0 \quad \text{--- (1)}$$

where $L_0 = \frac{R}{L} = \frac{1}{L} \int_0^{2\pi} R_0 d\phi = \frac{1}{L} \int_0^{2\pi} \frac{R_0}{\sin \theta} d\phi$

$$= \frac{R_0}{L \sin \theta} \int_0^{2\pi} d\phi = \frac{2\pi R_0}{L \sin \theta} \quad \text{--- (2)}$$

At $t=0$, $L_0(t) = L_0 \sin \theta$ is applied and (1) becomes

$$L \frac{dE_0}{dt} + R L_0 = L_{\text{enc}} L_0 E_0 \quad \text{--- (3)}$$

Solution of (3): $E_0 = \frac{E_0}{L} e^{-\frac{R L_0}{L} t}$, $0 < t < \tau$ --- (4)

At $t=\tau$, $L_0 = \frac{R_0}{L} e^{-\frac{R L_0}{L} \tau}$ when a negative step function $-L_0 \sin \theta$ is applied. If $\tau > L_0/L$, then L_0 for $t > \tau$ is the reverse of L_0 for $t < \tau$.

$$\text{For } t > \tau, L_0 = -\frac{R_0}{L} e^{-\frac{R L_0}{L} (t-\tau)}$$


At steady state, $dE_0/dt = 0$

$$0 = L \left(\frac{dE_0}{dt} \right) + R L_0 = \int_0^{2\pi} R_0 e^{-\frac{R L_0}{L} t} d\phi$$

$$= \frac{R_0}{L} (L_0 \tau) \quad \text{--- (5)}$$

Ex-23: $B = \mu_0 \frac{I}{2\pi r}$, $B = \mu_0 \frac{I}{2\pi r}$, $dr = \frac{r}{2\pi} d\theta$, $\theta = \frac{2\pi}{3} - \theta_0$

a) $B = \frac{\mu_0 I}{2\pi} \int_{\theta_0}^{\frac{2\pi}{3}} \frac{1}{r} dr = \frac{\mu_0 I}{2\pi} \ln \left(\frac{2}{1} \right) = \frac{\mu_0 I}{2\pi} \ln 2$

$\theta = \frac{\pi}{3}$, $r = \frac{2}{3} = \frac{2}{3} \ln \left(\frac{2}{1} \right) = \frac{2}{3} \ln 2$

$B = \frac{\mu_0 I}{2\pi} \ln 2 = \frac{\mu_0 I}{2\pi} \ln 2$



b) $I = \frac{1}{2} \left(\frac{1}{2} + \frac{1}{2} \right) = \frac{1}{2}$

$B = \frac{\mu_0 I}{2\pi} \ln 2 = \frac{\mu_0 I}{2\pi} \ln 2$

$B = \frac{\mu_0 I}{2\pi} \ln 2 = \frac{\mu_0 I}{2\pi} \ln 2$

Ex-24: From Problem 14-40, $B = \mu_0 I \frac{a}{2\pi r}$

a) $r = \frac{1}{2} = \frac{1}{2} \ln 2 = \frac{1}{2} \ln 2$

$I = \frac{1}{2} \ln 2 = \frac{1}{2} \ln 2$

$I = \frac{1}{2} \ln 2 = \frac{1}{2} \ln 2$

b) $I = \frac{1}{2} \ln 2 = \frac{1}{2} \ln 2$

Ex-25: a) Flux enclosed in the ring is $\Phi_B = \mu_0 I \frac{a}{2\pi r}$ ①

The induced EMF in the ring according to the assigned direction for current: $\mathcal{E} = \frac{d\Phi_B}{dt} = \frac{d}{dt} \left(\mu_0 I \frac{a}{2\pi r} \right)$ ②

Resistance of all three parallel rings: $R = \frac{3}{2} \frac{\rho L}{A}$ ③

Combining ① and ②: $I = \frac{\mathcal{E}}{R} = \frac{\mu_0 I \frac{a}{2\pi r}}{\frac{3}{2} \frac{\rho L}{A}}$ ④

$I = \frac{\mu_0 I \frac{a}{2\pi r}}{\frac{3}{2} \frac{\rho L}{A}}$ ⑤

$I = \frac{\mu_0 I \frac{a}{2\pi r}}{\frac{3}{2} \frac{\rho L}{A}}$ ⑥

$R = \frac{3}{2} \frac{\rho L}{A}$ ⑦

Ex. 2.10 a) $\rho_0 = 1 \cdot \lambda_0$, $\lambda_0 = 2\pi a \cdot 1 = 4\pi$ nm.

$$E = \frac{U}{\lambda_0} = \lambda_0 \cdot \frac{1}{4\pi} = 1 \text{ eV}; \quad E = \rho_0 \lambda_0 \cdot E = \lambda_0 \cdot \frac{1}{4\pi} \cdot \rho_0 \cdot \lambda_0$$

$$\begin{aligned} \text{b) } \psi_0 = \frac{1}{\sqrt{2}} \sin \frac{1}{2} \cdot x &= \frac{1}{\sqrt{2}} (\lambda_0 \cdot \omega) = \lambda_0 \cdot \omega = \lambda_0 \cdot \frac{1}{4\pi} \\ &= \frac{1}{\sqrt{2}} \sin \left(\frac{1}{4\pi} \cdot x \right) = \frac{1}{\sqrt{2}} \sin \left(\frac{1}{4\pi} \cdot \rho_0 \cdot \lambda_0 \cdot x \right) \end{aligned}$$

$$\begin{aligned} \text{c) } E_0 = \frac{U}{\lambda_0} = E \cdot \lambda_0 &= \frac{1}{\sqrt{2}} \cdot (\lambda_0 \cdot \omega) = (\lambda_0 \cdot \omega) = \lambda_0 \cdot \left(\frac{1}{4\pi} \cdot \rho_0 \cdot \lambda_0 \right) \\ \text{In direct analogy: } \psi &= \frac{1}{\sqrt{2}} \sin \omega \cdot x = \frac{1}{\sqrt{2}} \sin \left(\frac{1}{4\pi} \cdot \rho_0 \cdot \lambda_0 \cdot x \right) \\ &= \frac{1}{\sqrt{2}} \sin \frac{1}{4\pi} \end{aligned}$$

Short circuit: $\psi = 0$, $\lambda_0 = \frac{1}{\sqrt{2}} \sin \left(\frac{1}{4\pi} \right)$, where $\lambda_0 = \frac{1}{\sqrt{2}} \sin \frac{1}{4\pi}$.

Ex. 2.11 a) $E = -\nabla \psi = \frac{\partial \psi}{\partial x} = -\nabla \left(\psi \cdot \frac{1}{\sqrt{2}} \right) = \frac{\partial \psi}{\partial x} \rightarrow \psi = \psi \cdot \frac{1}{\sqrt{2}}$

$$\begin{aligned} \text{b) } E_y (x=0): \quad \psi \cdot E &= \rho_0 \cdot \frac{1}{\sqrt{2}} = 0 \\ \rightarrow \psi \cdot (E + \nabla \psi) &= \rho_0 \cdot \frac{1}{\sqrt{2}} \cdot (\psi + \frac{\partial \psi}{\partial x}) = 0 \\ \rightarrow \psi \cdot \rho_0 \cdot \frac{1}{\sqrt{2}} &= 0 \end{aligned}$$

Ex. 2.12 $E_y (x=0): \quad \psi \cdot E = \rho_0 \cdot \frac{1}{\sqrt{2}} \rightarrow \psi \cdot \left(\frac{1}{\sqrt{2}} \right) = \rho_0 \cdot \frac{1}{\sqrt{2}} \quad \textcircled{1}$

$E_y (x=0) \text{ to } x=a): \quad E = \rho_0 \cdot E, \quad E = -\nabla \psi = \frac{\partial \psi}{\partial x} \quad \textcircled{2}$

E -field along E_y axis: $\rho \cdot E = \left(\frac{1}{\sqrt{2}} \cdot E \right) \cdot \rho \cdot \frac{1}{\sqrt{2}} = \rho \cdot \frac{1}{\sqrt{2}} \cdot \rho \cdot \frac{1}{\sqrt{2}}$

Using gauge condition for potentials in an electromagnetic medium
 $\nabla \cdot (E \cdot \lambda_0) = \rho \cdot \frac{1}{\sqrt{2}} = 0$.

we obtain the wave equation for ψ : $\nabla \cdot \left(\frac{1}{\sqrt{2}} \cdot E \cdot \lambda_0 \right) = \rho \cdot \left(\frac{1}{\sqrt{2}} \cdot \rho \cdot \lambda_0 \right) = \frac{1}{\sqrt{2}} \cdot \rho \cdot \rho$

$E_y (x=0): \quad \psi \cdot E = \rho \rightarrow \psi \cdot \left(\frac{1}{\sqrt{2}} \cdot \rho \cdot \lambda_0 \right) = \rho \cdot \frac{1}{\sqrt{2}}$

Using equation for wave potential: $\frac{1}{\sqrt{2}} \cdot \psi \cdot \rho \cdot \lambda_0 = \rho \cdot \frac{1}{\sqrt{2}}$

Ex. 2.13 a) $E_y (x=0): \quad \psi \cdot E = \rho_0 \rightarrow \lambda_0 \cdot (\psi \cdot \frac{1}{\sqrt{2}}) = \rho_0$

$$\text{b) } \lambda_0 E = \rho \cdot \rho \quad \left\{ \begin{array}{l} \psi \cdot E = \rho \rightarrow \lambda_0 (\psi \cdot \frac{1}{\sqrt{2}}) = \rho \end{array} \right. \quad \textcircled{1}$$

$$\left\{ \begin{array}{l} \psi \cdot E = \rho \rightarrow \lambda_0 (\psi \cdot \frac{1}{\sqrt{2}}) = \rho \end{array} \right. \quad \textcircled{2}$$

Comparing $\lambda_0 \cdot (\psi \cdot \frac{1}{\sqrt{2}}) = \frac{1}{\sqrt{2}} \cdot (\lambda_0 \cdot \psi)$ Comparing $\lambda_0 \cdot \psi = \frac{1}{\sqrt{2}} \cdot (\lambda_0 \cdot \psi)$

$$\begin{aligned} \text{E.2.22} \quad \Phi \cdot \mathcal{E} &= -\mu \frac{\partial^2}{\partial x^2} \quad \text{①} & \Phi \cdot \mathcal{H} &= \mathcal{F} + c \frac{\partial^2}{\partial x^2} \quad \text{②} \\ \Phi \cdot \mathcal{E} &= \mathcal{F} \quad \text{③} & \Phi \cdot \mathcal{H} &= 0 \quad \text{④} \end{aligned}$$

$$\Phi \cdot \text{①}: \quad \Phi \cdot \Phi \cdot \mathcal{E} = -\mu \frac{\partial^2}{\partial x^2} (\Phi \cdot \mathcal{E}) = -\mu \frac{\partial^2}{\partial x^2} (\mathcal{F} + c \frac{\partial^2}{\partial x^2}) = -\Phi (\Phi \cdot \mathcal{E}) = -\Phi \mathcal{F}$$

Wave equation for $\mathcal{F}$: $\Phi^2 \mathcal{F} - \mu \frac{\partial^2}{\partial x^2} \mathcal{F} = \mu \frac{\partial^2}{\partial x^2} \mathcal{F} = -\frac{1}{2} \Phi \mathcal{F}$

$$\Phi \cdot \text{②}: \quad \Phi \cdot \Phi \cdot \mathcal{H} = \Phi \cdot \mathcal{F} + c \frac{\partial^2}{\partial x^2} (\Phi \cdot \mathcal{H}) = \Phi \cdot \mathcal{F} + c \mu \frac{\partial^2}{\partial x^2} \mathcal{F} = \Phi (\Phi \cdot \mathcal{H}) = \Phi \cdot 0$$

Wave equation for $\mathcal{F}$: $\Phi^2 \mathcal{F} - \mu \frac{\partial^2}{\partial x^2} \mathcal{F} = -\Phi \cdot \mathcal{F}$

For identical time dependence: $\frac{\partial}{\partial t} \rightarrow -i\omega$, $\frac{\partial^2}{\partial t^2} \rightarrow -\omega^2$

Helmholtz equation: $\Phi^2 \mathcal{E} + \omega^2 \mu \mathcal{E} = -\mu \omega^2 \mathcal{E} = -\frac{1}{2} \Phi \mathcal{E}$

(for photon) $\Phi^2 \mathcal{H} + \omega^2 \mu \mathcal{H} = -\Phi \cdot \mathcal{H}$

E.2.23 $\mathcal{E} = \mathcal{E}_0 \sin(kx) \cos(\omega t) = \mu \mathcal{H}$ (12.1)

Use phasors: $\tilde{\mathcal{H}} = \frac{1}{\mu} \frac{\partial}{\partial x} \tilde{\mathcal{E}} = \frac{1}{\mu} \frac{\partial}{\partial x} [\mathcal{E}_0 \sin(kx) \cos(\omega t) e^{-i\omega t}]$

$$\mathcal{E} = \frac{1}{\mu} \frac{\partial}{\partial x} \Phi \cdot \tilde{\mathcal{H}} = \mathcal{E}_0 \frac{\partial}{\partial x} \left[\frac{1}{\mu} \cos(kx) \right] \cos(\omega t) = \mathcal{E}_0 \cos(kx) \cos(\omega t) \quad \text{①}$$

Again from $\mathcal{E} = \mathcal{E}_0 \sin(kx) \cos(\omega t)$ (12.1) ②

Substituting ① and ②: $(\cos^2(kx) + \sin^2(kx)) \mu \mathcal{E}_0 \cos(\omega t) = \mu \mathcal{E}_0 \cos(\omega t)$
 $\Rightarrow \mu \mathcal{E}_0 \cos(\omega t) = \mu \mathcal{E}_0 \cos(\omega t)$

From ②: $\tilde{\mathcal{H}} = \frac{1}{\mu} \frac{\partial}{\partial x} (\Phi \mathcal{E}_0 \cos(kx))$
 $= -\frac{\mathcal{E}_0}{\mu} \sin(kx) \cos(\omega t) = \mathcal{H}_0 \sin(kx) \cos(\omega t)$
 $\Rightarrow \mathcal{H}_0 = \frac{\mathcal{E}_0}{\mu} \sin(kx) \cos(\omega t) = \mathcal{H}_0 \sin(kx) \cos(\omega t)$ (12.2)

E.2.24 $\mathcal{E}(x,t) = \mathcal{E}_0 \sin(kx) \cos(\omega t) = \mu \mathcal{H}$ (12.1)

Phasors: $\tilde{\mathcal{H}} = \frac{1}{\mu} \frac{\partial}{\partial x} \tilde{\mathcal{E}}$

Substituting the Phasors: $(\cos^2(kx) + \sin^2(kx)) \mu \mathcal{E}_0 \cos(\omega t) = \mu \mathcal{E}_0 \cos(\omega t)$
 $\Rightarrow \mu \mathcal{E}_0 \cos(\omega t) = \mu \mathcal{E}_0 \cos(\omega t)$

$$\mathcal{E} = \frac{1}{\mu} \frac{\partial}{\partial x} \Phi \cdot \tilde{\mathcal{H}} = \left[\frac{\mathcal{E}_0}{\mu} \sin(kx) \cos(\omega t) + \frac{\mathcal{E}_0}{\mu} \sin(kx) \cos(\omega t) \right] \mu \cos(\omega t)$$

$$\mathcal{E}(x,t) = \mathcal{E}_0 \sin(kx) \cos(\omega t) = \mathcal{E}_0 \sin(kx) \cos(\omega t) = \mathcal{E}_0 \sin(kx) \cos(\omega t) = \mathcal{E}_0 \sin(kx) \cos(\omega t)$$

 $\Rightarrow \mathcal{E}_0 \sin(kx) \cos(\omega t) = \mathcal{E}_0 \sin(kx) \cos(\omega t)$ (12.2)

$$\begin{aligned}
 \text{E2.12 Use plane: } E &= E_0 \frac{d}{dt} \cos \theta = -\dot{\theta} E_0 \sin \theta \\
 \vec{v} \times \vec{E} &= E_0 \frac{1}{c} \frac{d\theta}{dt} (E_0 \sin \theta) = E_0 \sin \theta \frac{d\theta}{dt} \cos \theta = \dot{\theta} E_0 \sin \theta \cos \theta \\
 &= j \omega E_0 \vec{B} \rightarrow \vec{B} = E_0 \frac{1}{j\omega} \sin \theta \cos \theta = E_0 \frac{1}{2j\omega} \sin 2\theta
 \end{aligned}$$

$$\begin{aligned}
 \text{In free space, } E &= E_0 \sin \theta \rightarrow B = E_0 \frac{1}{2j\omega} \sin 2\theta = \frac{E_0}{2j\omega} \sin 2\theta \\
 \vec{B} \times E_0 \sin \theta &= E_0 \frac{1}{2j\omega} \sin 2\theta \sin \theta \cos \theta = \frac{E_0}{4j\omega} \sin 4\theta
 \end{aligned}$$

$$\begin{aligned}
 \text{E2.13 Maxwell's equations } \vec{v} \times \vec{B} &= -j\omega \mu \vec{D} \quad (1) \\
 \vec{v} \times \vec{D} &= j\omega \epsilon \vec{B} \quad (2)
 \end{aligned}$$

$$\text{From } \vec{v} \times \vec{B} = 0, \text{ define } \vec{A}_1 \text{ such that } \vec{B} = \vec{v} \times \vec{A}_1 \quad (3)$$

$$\begin{aligned}
 \text{From (2), } \vec{B} &= \frac{1}{j\omega \epsilon} \vec{v} \times \vec{D} = \frac{1}{j\omega \epsilon} \vec{v} \times \vec{v} \times \vec{A}_2 \\
 &= \frac{1}{j\omega \epsilon} [\vec{v} (\vec{v} \cdot \vec{A}_2) - \vec{v}^2 \vec{A}_2] \quad (4)
 \end{aligned}$$

$$\text{From (3), } \vec{v} \times (\vec{v} \times \vec{A}_1) = \vec{B} \text{ let } \vec{B} = j\omega \mu \vec{A}_2 = \vec{v} \times \vec{A}_1 \quad (5)$$

$$\text{Substituting (4) from (5): } \omega \epsilon \vec{A}_2 = \frac{1}{j\omega \epsilon} [\vec{v} (\vec{v} \cdot \vec{A}_2) - \vec{v}^2 \vec{A}_2] - j\omega \mu \vec{A}_2$$

$$\text{Choose } \vec{v} \cdot \vec{A}_2 = j\omega \mu \epsilon \vec{A}_2 \quad (6)$$

$$\text{a) If } \epsilon \text{ decreases } \vec{B} = j\omega \mu \vec{A}_2 = \frac{1}{j\omega \epsilon} \vec{v} \times (\vec{v} \cdot \vec{A}_2)$$

$$\text{b) If } \epsilon \text{ decreases } \vec{v} \cdot \vec{A}_2 = j\omega \mu \epsilon \vec{A}_2 \text{ will no longer be satisfied by}$$

$$\text{E2.14 } \vec{B} = j\omega \mu \vec{v} \times \vec{E} \quad (1)$$

$$\begin{aligned}
 \vec{v} \times \vec{E} &= -j\omega \mu \vec{B} = -j\omega \mu \vec{v} \times \vec{E}_1 \\
 \Rightarrow \vec{v} \cdot (\vec{E} - \vec{E}_1) \vec{v} &= 0 \text{ let } \vec{E} = \vec{E}_1 + \vec{v} \times \vec{E}_2 \quad (2)
 \end{aligned}$$

$$\vec{v} \times \vec{E} = j\omega \mu \vec{v} \times (\vec{E}_1 + \vec{v} \times \vec{E}_2) = j\omega \mu (\vec{E}_1 + \vec{v} \times \vec{E}_2) \quad (3)$$

$$\text{Substituting (2) and (3) in (1):}$$

$$\begin{aligned}
 j\omega \mu \vec{v} \times \vec{v} \times \vec{E}_2 &= j\omega \mu (\vec{E}_1 + \vec{v} \times \vec{E}_2 + \vec{v} \times \vec{v} \times \vec{E}_2) \\
 &= j\omega \mu (\vec{v} \times \vec{E}_2 - \vec{v}^2 \vec{E}_2) \quad (4)
 \end{aligned}$$

$$\text{Choose } \vec{v} \cdot \vec{E}_2 = 0 \text{ If } \epsilon \text{ decreases}$$

$$\text{b) } \vec{v} \cdot \vec{E}_2 = \vec{E}_2 \cdot \vec{E}_2 = -\frac{E^2}{\epsilon} \quad (5)$$

(i) $E_2 \oplus$ becomes

$$\begin{aligned} E &= E_1^* E_2 + \sigma \sigma^* E_2 \\ &= E_1^* E_2 + (\sigma^* E_2 + \sigma \sigma^* E_2). \end{aligned} \quad (2)$$

Combination of Eqs. (1), (2) and (3) gives

$$E = \sigma \sigma^* \sigma^* E_2 = \frac{E}{\gamma^2}.$$

2.2.22

$$\begin{aligned} (i) \quad \left| \frac{\text{Refractive index}}{\text{Conductivity}} \right| &= \frac{\sigma \mu}{\sigma} = \frac{(\sigma \mu \cos \theta) \sin \theta}{\sigma \cos \theta} \\ &= \mu \sin^2 \theta. \end{aligned}$$

(ii) In a source-free conductor:

$$\nabla \times \vec{E} = -\sigma \vec{E}, \quad (3)$$

$$\vec{E} = \vec{E} = -\text{grad} \vec{V}, \quad (4)$$

$$\nabla \times \vec{E} = \sigma \nabla \times \vec{E} = \sigma (\nabla \times \vec{E} - \sigma^* \vec{E} - \sigma \nabla \times \vec{E}). \quad (5)$$

Since $\nabla \times \vec{E} = 0$, $E_2 \oplus$ becomes

$$\sigma^* \vec{E} = \sigma \nabla \times \vec{E} = 0 \quad (6)$$

Combining (3) and (6)

$$\sigma^* \vec{E} = -\text{grad} \sigma \vec{E} = 0.$$

Chapter 8

Plane Electromagnetic Waves

Ex.1 In a source-free simple medium,

$$\vec{E}_2(z=0,t) = \vec{E} + \vec{F} = E_0 \hat{x} + F_0 \hat{y} + e^{-j\omega t} \hat{z} \quad (1)$$

$$\vec{H}_2(z=0,t) = \vec{H} + \vec{F} = H_0 \hat{x} - F_0 \hat{y} - j\omega \frac{\mu}{k} (E_0 \hat{y} - F_0 \hat{x}) \quad (2)$$

Substituting (1) in (2) and noting that $\vec{F} \cdot \vec{E} = 0$:

$$F_0^2 = j\omega \mu \frac{E_0^2}{k} - j\omega \mu \frac{F_0^2}{k} = 0$$

Similarly for $\vec{E}$.

Ex.2 Assume that the vehicle moves with a velocity u in the z direction, which is the direction of propagation of the incident wave.

$$a) \quad \vec{E}_i = E_0 \hat{x} e^{j(\omega t - kz)} \quad \vec{E}_r = -E_0 \hat{x} e^{j(\omega t + kz)}$$

$\vec{E}_i + \vec{E}_r = 0$ must be satisfied on reflecting surface for all t
 $z = 0$: $(\omega t - kz) = (\omega t + kz)$

$$\rightarrow \omega t - kz = (\omega t + kz) \Rightarrow \omega = \left(\frac{\omega}{k} + \frac{\omega}{k}\right) \Rightarrow \omega = (\omega + u) \frac{\omega}{k}$$

$$\rightarrow \frac{\omega^2}{k} = 1 - \frac{\omega}{k} \left(1 + \frac{\omega}{k}\right)$$

$$\rightarrow \frac{\omega}{k} = \frac{1}{2} + \frac{1}{2} \frac{\omega^2}{k^2} = 1 - \frac{1}{2} \frac{u^2}{c^2} \quad \text{for } u \ll c$$

$$\rightarrow \omega f = f - f = -\frac{1}{2} \frac{u^2}{c^2} f$$

b) For $\omega f = -0.11 \text{ rev}^2/\text{day}$ and $f = 60 \text{ rev}^2/\text{day}$ (corresponding to 1 Hz)

Ex.3 Maximum time dependence: $e^{j(\omega t - kz)}$ $\frac{\partial}{\partial t} = -j\omega$

Maxwell: $\vec{E} = E_0 e^{j(\omega t - kz)} \quad \vec{H} = H_0 e^{j(\omega t - kz)}$ where E_0 and H_0 are constant vectors.

$$\text{Now: } \vec{\nabla} \cdot (\rho e^{j(\omega t - kz)}) = \rho e^{j(\omega t - kz)} \vec{\nabla} \cdot \vec{E} = \rho e^{j(\omega t - kz)} \left(\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} \right)$$

$$= \rho E_0 \hat{x} \cdot \hat{x} + 0 + 0 = \rho E_0 e^{j(\omega t - kz)} = \rho E_0 e^{j(\omega t - kz)}$$

Maxwell equation: $\vec{\nabla} \cdot \vec{E} = \rho / \epsilon_0 \Rightarrow \rho = \epsilon_0 \vec{\nabla} \cdot \vec{E} \Rightarrow E_0 = \epsilon_0 \rho$

$$\rho = E_0 / \epsilon_0 \Rightarrow \rho e^{j(\omega t - kz)} = \frac{E_0}{\epsilon_0} e^{j(\omega t - kz)} \Rightarrow E_0 = \epsilon_0 \rho$$

$$\rho \cdot E = \rho e^{j(\omega t - kz)} E_0 = \rho \cdot E = \epsilon_0 \rho \cdot E \Rightarrow \rho \cdot E = \epsilon_0 \rho \cdot E \Rightarrow \rho \cdot E = \epsilon_0 \rho \cdot E$$

Ex. 10.1.1 $\vec{r} = \vec{r}_0 + \omega r^2 \cos(\omega^2 t - \frac{1}{2}\pi) \hat{r}$ (10.1.1)

(i) $\dot{r}_0 = \omega r_0^2 \frac{d}{dt} \cos(\omega^2 t - \frac{1}{2}\pi) = -\frac{2}{3}\omega^2 r_0^2 \sin(\omega^2 t - \frac{1}{2}\pi)$

$\dot{r}_0 = -\frac{2}{3}\omega^2 r_0^2 \sin(\omega^2 t - \frac{1}{2}\pi)$

At $t = 0, \omega^2 t = 0$, we require the argument of cosine to be 0

$\omega^2 t - \frac{1}{2}\pi = 0 \Rightarrow \omega^2 t = \frac{1}{2}\pi = \omega^2 t_0 \Rightarrow t_0 = \frac{\pi}{2\omega^2}$

$\Rightarrow \vec{r} = \vec{r}_0 + \omega r^2 \cos(\omega^2 t - \frac{1}{2}\pi) = \vec{r}_0 + \omega r_0^2 \cos(\omega^2 t - \frac{1}{2}\pi)$

Ex. 10.1.2 Answer: $\vec{r} = \vec{r}_0 + \omega r_0^2 \cos(\omega^2 t) \hat{r}$ (10.1.2)

(i) $\omega = \omega^2 \cos(\omega^2 t) \Rightarrow \dot{r} = \omega^2 \dot{r}_0 = \omega^2 \omega^2 \cos(\omega^2 t)$

$\dot{r} = \omega^2 \cos(\omega^2 t) \Rightarrow \dot{r} = \omega^2 \cos(\omega^2 t)$

(ii) $\dot{r} = \frac{dr}{dt} = \frac{dr}{d\theta} \frac{d\theta}{dt} \Rightarrow \dot{r} = \left(\frac{dr}{d\theta}\right) \dot{\theta}$

(iii) $\dot{r} = \omega^2 \cos(\omega^2 t)$

(iv) $\dot{r} = \omega^2 \cos(\omega^2 t) = \frac{dr}{d\theta} \frac{d\theta}{dt} = \frac{dr}{d\theta} \omega^2 \cos(\omega^2 t)$

$\dot{r} = \frac{dr}{d\theta} \omega^2 \cos(\omega^2 t) = \frac{dr}{d\theta} \omega^2 \cos(\omega^2 t)$

$\dot{r} = \frac{dr}{d\theta} \omega^2 \cos(\omega^2 t) = \frac{dr}{d\theta} \omega^2 \cos(\omega^2 t)$ (10.1.3)

Ex. 10.1.3 Let $\vec{r} = r \hat{r}$, $\vec{r} = r_0 + \omega r_0^2 \cos(\omega^2 t) \hat{r}$ (10.1.4)

$\frac{dr}{dt} = \omega^2 \cos(\omega^2 t)$, $\frac{dr}{d\theta} = \frac{dr}{d\theta} \frac{d\theta}{dt} = \frac{dr}{d\theta} \omega^2 \cos(\omega^2 t)$

$\left(\frac{dr}{d\theta} - \frac{dr}{d\theta} \cos(\omega^2 t)\right) = \left(1 - \frac{dr}{d\theta}\right) \cos(\omega^2 t)$

$\left(\frac{dr}{d\theta} - \frac{dr}{d\theta} \cos(\omega^2 t)\right) = \left(1 - \frac{dr}{d\theta}\right) \cos(\omega^2 t)$ (10.1.5)

which is the equation of an ellipse. In order to find the



parameters of the particular ellipse, note the eccentricity does not depend on the angle θ or $\omega^2 t$. Assume the equation of the ellipse is then

$\left(\frac{r}{a}\right)^2 + \left(\frac{r}{b}\right)^2 = 1$ (10.1.6)

$$\text{where } E_x = E_0 \cos \theta + E_y \sin \theta, \quad (2)$$

$$\text{and } E_y = -E_0 \sin \theta + E_y \cos \theta. \quad (3)$$

Substituting (2) and (3) in (1) and rearranging

$$E_0^2 \left(\frac{\cos^2 \theta}{\epsilon_1^2} + \frac{\sin^2 \theta}{\epsilon_2^2} \right) + E_y^2 \left(\frac{\sin^2 \theta}{\epsilon_1^2} + \frac{\cos^2 \theta}{\epsilon_2^2} \right) - 2E_0 E_y \sin \theta \cos \theta \left(\frac{1}{\epsilon_1} - \frac{1}{\epsilon_2} \right) = 1. \quad (4)$$

Comparing (4) and (3), we obtain

$$\begin{cases} \frac{\cos^2 \theta}{\epsilon_1^2} + \frac{\sin^2 \theta}{\epsilon_2^2} = \frac{1}{E_0^2 \cos^2 \theta}, \\ \frac{\sin^2 \theta}{\epsilon_1^2} + \frac{\cos^2 \theta}{\epsilon_2^2} = \frac{1}{E_0^2 \sin^2 \theta}, \\ \sin \theta \cos \theta \left(\frac{1}{\epsilon_1} - \frac{1}{\epsilon_2} \right) = \frac{E_y}{E_0^2 \sin 2\theta}. \end{cases} \quad (5)$$

$$\begin{cases} \frac{\cos^2 \theta}{\epsilon_1^2} + \frac{\sin^2 \theta}{\epsilon_2^2} = \frac{1}{E_0^2 \cos^2 \theta}, \\ \frac{\sin^2 \theta}{\epsilon_1^2} + \frac{\cos^2 \theta}{\epsilon_2^2} = \frac{1}{E_0^2 \sin^2 \theta}, \end{cases} \quad (6)$$

$$\sin \theta \cos \theta \left(\frac{1}{\epsilon_1} - \frac{1}{\epsilon_2} \right) = \frac{E_y}{E_0^2 \sin 2\theta}. \quad (7)$$

Eqn. (5), (6) and (7) can be solved for three unknowns:

$$\theta = \frac{1}{2} \tan^{-1} \left(\frac{4E_y \epsilon_1 \epsilon_2 \sin 2\theta}{E_0^2 (\epsilon_1^2 - \epsilon_2^2)} \right),$$

$$a = \sqrt{\frac{1}{\epsilon_1^2 \cos^2 \theta + \epsilon_2^2 \sin^2 \theta} - \frac{1}{E_0^2 \cos^2 \theta}}, \quad \sin \theta,$$

$$b = \sqrt{\frac{1}{\epsilon_1^2 \sin^2 \theta + \epsilon_2^2 \cos^2 \theta} - \frac{1}{E_0^2 \sin^2 \theta}}, \quad \cos \theta.$$

In particular, if $E_x = E_0 + E_y$, $\theta = 45^\circ$, $a = \sqrt{\epsilon_1^2 - \epsilon_2^2}/E_0$, $b = \sqrt{\epsilon_2^2 - \epsilon_1^2}/E_0$.

2.2.2. Let an elliptically polarized plane wave be represented by the phasor (with propagation factor e^{ikz} omitted):

$$\vec{E} = E_0 \hat{x} + E_y \hat{y} e^{i\phi},$$

where E_0 , E_y and ϕ are arbitrary constants.

Right-hand circularly polarized wave: $E_y = E_0 (\hat{x} - i\hat{y})$

Left-hand circularly polarized wave: $E_y = E_0 (\hat{x} + i\hat{y})$

$$\text{If } E_0 = \frac{1}{2}(\hat{x} + i\hat{y}) e^{i\phi} \text{ and } E_y = \frac{1}{2}(\hat{x} + i\hat{y}) e^{i\phi},$$

$$\text{then } \vec{E} = \vec{E}_R + \vec{E}_L.$$

$$\text{a) Right-hand circularly polarized wave: } \vec{E}_R = \frac{1}{2}(\hat{x} - i\hat{y}) e^{i\phi}$$

$$= \frac{1}{2}(\hat{x} \hat{x} + \hat{y} \hat{y}) + \frac{1}{2}(\hat{x} \hat{y} - \hat{y} \hat{x})$$

$$= \vec{E}_R + \vec{E}_L, \quad \text{where } \vec{E}_R \text{ and } \vec{E}_L \text{ are}$$

right-hand and left-hand elliptically polarized waves respectively.

$$\text{b) Left-hand circularly polarized wave: } \vec{E}_L = \frac{1}{2}(\hat{x} + i\hat{y}) e^{i\phi} = \frac{1}{2}(\hat{x} \hat{x} + \hat{y} \hat{y}) + \frac{1}{2}(\hat{x} \hat{y} + \hat{y} \hat{x})$$

$$= \vec{E}_R + \vec{E}_L.$$

Ex-1 For conducting medium: $k_2 = \beta - j\alpha$.

$$\begin{aligned} \lambda_2^2 &= \beta^2 - \alpha^2 - 2j\alpha\beta \\ &= \alpha^2 \mu \epsilon_2 = \alpha^2 \mu \epsilon_2 (1 - j \frac{\sigma}{\omega \epsilon_2}) \end{aligned}$$

$$\therefore \beta^2 - \alpha^2 = \alpha^2 / \lambda_2^2 = \alpha^2 \mu \epsilon_2 \quad \text{--- (1)}$$

$$\beta + \alpha^2 = |\lambda_2^2| = \alpha^2 \mu \epsilon_2 \sqrt{1 + \left(\frac{\sigma}{\omega \epsilon_2}\right)^2} \quad \text{--- (2)}$$

From (1) and (2) we obtain

$$\alpha = \alpha \sqrt{\mu \epsilon_2} \left[\sqrt{1 + \left(\frac{\sigma}{\omega \epsilon_2}\right)^2} - 1 \right]^{1/2}, \quad \beta = \alpha \sqrt{\mu \epsilon_2} \left[\sqrt{1 + \left(\frac{\sigma}{\omega \epsilon_2}\right)^2} + 1 \right]^{1/2}$$

Ex-2 All three materials are good conductors, $\left(\frac{\sigma}{\omega \epsilon_2}\right)^2 \gg 1$.

$$\alpha = \omega \sqrt{\mu \epsilon_2} \sqrt{\sigma}, \quad \beta = \frac{\sigma}{2}, \quad \gamma_2 = (1+j) \sqrt{\frac{\sigma \mu \omega}{2}}$$

a) $f = 4\pi \times 10^6$

	γ_1 (dB)	α (dB/m)	ω (dB/m)	β (dB)
Copper	$1.815 \times 10^4 \text{ dB/m}$	$8.48 \times 10^3 \text{ dB/m}$	1.04×10^3	8.17×10^3
Silver	$1.815 \times 10^4 \text{ dB/m}$	$8.48 \times 10^3 \text{ dB/m}$	1.04×10^3	8.17×10^3
Alum.	$1.815 \times 10^4 \text{ dB/m}$	$8.48 \times 10^3 \text{ dB/m}$	1.04×10^3	8.17×10^3

b) $f = 1 \text{ GHz}$

	γ_1 (dB)	α (dB/m)	ω (dB/m)	β (dB)
Copper	$8.48 \times 10^4 \text{ dB/m}$	$4.24 \times 10^4 \text{ dB/m}$	$5.2 \times 10^3 \text{ dB/m}$	$4.08 \times 10^4 \text{ dB/m}$
Silver	$8.48 \times 10^4 \text{ dB/m}$	$4.24 \times 10^4 \text{ dB/m}$	$5.2 \times 10^3 \text{ dB/m}$	$4.08 \times 10^4 \text{ dB/m}$
Alum.	$8.48 \times 10^4 \text{ dB/m}$	$4.24 \times 10^4 \text{ dB/m}$	$5.2 \times 10^3 \text{ dB/m}$	$4.08 \times 10^4 \text{ dB/m}$

c) $f = 1 \text{ THz}$

	γ_1 (dB)	α (dB/m)	ω (dB/m)	β (dB)
Copper	$1.815 \times 10^5 \text{ dB/m}$	$8.48 \times 10^4 \text{ dB/m}$	$1.04 \times 10^4 \text{ dB/m}$	$8.17 \times 10^4 \text{ dB/m}$
Silver	$1.815 \times 10^5 \text{ dB/m}$	$8.48 \times 10^4 \text{ dB/m}$	$1.04 \times 10^4 \text{ dB/m}$	$8.17 \times 10^4 \text{ dB/m}$
Alum.	$1.815 \times 10^5 \text{ dB/m}$	$8.48 \times 10^4 \text{ dB/m}$	$1.04 \times 10^4 \text{ dB/m}$	$8.17 \times 10^4 \text{ dB/m}$

$$\text{Üb. 11.11 } f = \sin \pi^2 (2x), \quad \eta = 1, \delta, \quad \tan \xi = \frac{f}{\eta} = \pi^2$$

$$\text{a) } E_p (0 \rightarrow \pi/2): \quad \omega = \frac{1}{2} \sqrt{\frac{2}{3}} = \frac{1}{2} \left(\frac{2}{3} \right)^{1/2} = 0,408 \text{ (rad/s)},$$

$$\omega^{(0)} = \frac{1}{2} \longrightarrow \omega = \frac{1}{2} \text{ bis } 2 = 0,398 \text{ (rad/s)}$$

$$\text{b) } E_p (0 \rightarrow \pi/2): \quad \eta_1 = \frac{1}{2} \sqrt{\frac{2}{3}} (1 + \frac{1}{2} \frac{f}{\eta_1}) = 0,398 + 0,408 \sin \pi^2 \cos \pi^2 \sin \pi^2 \sin \pi^2$$

$$E_p (0 \rightarrow \pi/2): \quad \beta = \frac{1}{2} \sqrt{\frac{2}{3}} \left(1 + \frac{1}{2} \left(\frac{f}{\eta} \right)^2 \right) = 0,408 \text{ (rad/s)},$$

$$\omega = \frac{1}{2} = 0,398 \text{ (rad/s)},$$

$$\omega_p = \frac{f}{\eta} = 0,398 \sin \pi^2 \cos \pi^2 \sin \pi^2 \sin \pi^2$$

$$\omega_0 = \frac{1}{2} = \frac{1}{2} \left(1 + \frac{1}{2} \left(\frac{f}{\eta} \right)^2 \right) = 0,398 \sin \pi^2 \cos \pi^2 \sin \pi^2 \sin \pi^2$$

$$\text{c) } R(\pi) = 0, \quad E = E_p e^{j\pi\omega}$$

$$\tilde{E} = \frac{1}{\sqrt{2}} E_p = E = E_p \cos \pi\omega e^{j(\pi\omega - \pi/2 + \pi\omega)}$$

$$R(\pi\omega) = \tilde{E}_p \cos \pi\omega e^{j\pi\omega} \sin(\pi\omega\omega - \pi/2 + \pi\omega\omega) \text{ (rad/s)}$$

$$\text{Üb. 11.12 } \eta/\tan \xi = \eta/\sin \pi \sin \pi \left(\frac{f}{\eta} \cos \pi \right) = 0,39$$

$$\text{a) } \omega = \frac{1}{2} \sqrt{\frac{2}{3}} \left(\sqrt{1 + \left(\frac{2f}{\eta} \right)^2} - 1 \right) = 0,4 \text{ (rad/s)},$$

$$\beta = \frac{1}{2} \sqrt{\frac{2}{3}} \left(\sqrt{1 + \left(\frac{2f}{\eta} \right)^2} + 1 \right) = 0,398 \text{ (rad/s)},$$

$$\eta_1 = \frac{1}{2} \sqrt{\frac{2}{3}} = \frac{1}{2} \sqrt{\frac{2}{3}} \left(\frac{f}{\eta_1} \right) e^{j\pi\omega \sin \pi \cos \pi} = 0,398 \sin \pi \cos \pi$$

$$\omega_p = \omega_0 = 0,398 \sin \pi \cos \pi, \quad \omega_0 \sin \pi \cos \pi = 0,398 \sin \pi \cos \pi, \quad \beta = \frac{1}{2} \sqrt{\frac{2}{3}} \cos \pi$$

$$\text{b) } \omega^{(0)} = \frac{1}{2}, \quad \omega = \frac{1}{2} \text{ bis } 2 = 0,398 \text{ (rad/s)}$$

$$\text{c) } R(\pi, \omega) = \tilde{E}_p \cos \pi \omega e^{j\pi\omega \sin \pi \cos \pi} \sin(\pi\omega \sin \pi \cos \pi - \pi/2 + \pi\omega \sin \pi \cos \pi)$$

$$= \tilde{E}_p \cos \pi \omega e^{j\pi\omega \sin \pi \cos \pi} \sin(\pi\omega \sin \pi \cos \pi - \pi/2 + \pi\omega \sin \pi \cos \pi) \text{ (rad/s)}$$

$$E(\pi, \omega) = \tilde{E}_p \left(\tilde{E}_p \cos \pi \omega e^{j\pi\omega \sin \pi \cos \pi} \tilde{E}_p \cos \pi \omega e^{j\pi\omega \sin \pi \cos \pi} - \frac{1}{2} \cos \pi \omega \right)$$

$$\text{Üb. 11.13 a) } \beta = \frac{1}{2} \sqrt{\frac{2}{3}} \longrightarrow \omega = \frac{1}{2} \sqrt{\frac{2}{3}} = 0,398 \text{ (rad/s)}$$

$$\text{b) } R(\pi) = f = \pi^2 (2x), \quad \omega = \sqrt{\frac{2}{3}} \pi^2 = 0,398 \pi^2 \text{ (rad/s)}$$

$$\beta = \frac{1}{2} \sqrt{\frac{2}{3}} = 0,398 \pi^2 \longrightarrow \omega = \frac{1}{2} \sqrt{\frac{2}{3}} = 0,398 \pi^2 \text{ (rad/s)}$$

Ex. 11.11



Assume the interface to be stratified into layers having constant thickness

$$d_1 \ll d_2 \ll \dots \ll c = c N_{\text{max}}$$

The corresponding equivalent permittivities of the layers are:

$$\epsilon_1 = \epsilon_0 \left(1 - \frac{d_1}{c} \right) \quad \text{with} \quad \epsilon_{pm} = \frac{1}{\epsilon_0} \frac{d_1}{d_1 + d_2}$$

$$\text{and} \quad \epsilon_2 = \epsilon_0 \left(1 - \frac{d_2}{c} \right) \geq \epsilon_1 \geq \epsilon_2 \geq \dots \geq \epsilon_{N_{\text{max}}} \left(\frac{d_1}{d_1 + d_2} \right)$$

From Snell's law of refraction:

$$\sin \theta_i = \sin \theta_1 \sqrt{\epsilon_1 / \epsilon_0} = \sin \theta_1 \sqrt{1 - d_1 / c}$$

$$\sin \theta_1 = \sin \theta_2 \sqrt{\epsilon_2 / \epsilon_1} = \sin \theta_2 \sqrt{d_1 / (d_1 + d_2)}$$

$$\sin \theta_2 = \sin \theta_3 \sqrt{\epsilon_3 / \epsilon_2} = \sin \theta_3 \sqrt{d_2 / (d_1 + d_2)}$$

For total reflection at the layer with $\epsilon_{N_{\text{max}}}$, the angle of refraction $\theta_{N_{\text{max}}} = \pi/2$, and $\sin \theta_{N_{\text{max}}} = 1 = \sin \theta_1 \sqrt{\epsilon_1 / \epsilon_0}$

$$\therefore \quad \theta_{N_{\text{max}}} = \theta_1 \left(1 - \frac{d_1}{c} \right)^{1/2} = \theta_1 \sin \theta_1$$

$$\therefore \quad \theta = \theta_{N_{\text{max}}} / \cos \theta = \theta_1 \sqrt{\epsilon_1 / \epsilon_0} / \cos \theta_1$$

Ex. 11.12 a) From Eq. (11.72): $\epsilon_2 = \frac{c^2}{\omega^2} = \frac{c^2}{\omega_p^2} (\rho n_p) = n_p \rho \frac{c^2}{\omega_p^2}$

$$\text{b) } \epsilon = \frac{c^2}{\omega^2} \cdot \frac{\omega_p^2}{c^2} = \frac{\omega_p^2}{\omega^2} = \frac{1}{\beta^2}$$

$$n_p = n_p \rho \left(\frac{c^2}{\omega_p^2} - \frac{c^2}{\omega^2} \right) = n_p \rho \frac{c^2}{\omega^2}$$

Ex. 11.13 $\epsilon_{\text{eff}} = \frac{1}{2} (\epsilon_1 + \epsilon_2) = \frac{1}{2} (\epsilon_0 \sin \theta)$

a) $|\epsilon| = \frac{1}{2} \sqrt{\epsilon_1^2 + \epsilon_2^2} = 2.78 \times 10^{-10} \text{ F/m}$

b) $|\epsilon| = \frac{1}{2} \sqrt{\epsilon_1^2 + \epsilon_2^2} = 2.78 \times 10^{-10} \text{ F/m}$

c) $\epsilon_{\text{eff}} = \frac{1}{2} (\epsilon_1 + \epsilon_2) = 1.39 \times 10^{-10} \text{ F/m}$

$|\epsilon| = 1.39 \times 10^{-10} \text{ F/m}$, $|\epsilon| = 1.39 \times 10^{-10} \text{ F/m}$

Ex. 11 Assume a uniformly polarized plane sheet:

$$\vec{E}(x, y) = E_0 \vec{e}_y \cos(\omega t - k_0 y + \phi) + \vec{e}_y E_0 \sin(\omega t - k_0 y + \phi),$$

$$\vec{H}(x, y) = E_0 \frac{1}{c} \hat{y} \cos(\omega t - k_0 y + \phi) - E_0 \frac{1}{c} \hat{x} \sin(\omega t - k_0 y + \phi).$$

$$\begin{aligned} \text{Poynting vector, } \vec{S} = \vec{E} \times \vec{H} &= E_0 \frac{1}{c} \left[\cos^2(\omega t - k_0 y + \phi) \hat{z} - \sin^2(\omega t - k_0 y + \phi) \hat{z} \right] \\ &= E_0 \frac{1}{c} \hat{z}, \text{ a constant independent of } t \text{ and } y. \end{aligned}$$

Ex. 12 $\vec{E} = E_0 \vec{e}_x + E_0 \vec{e}_y$,

$$\vec{H} = \frac{1}{c} E_0 \vec{e}_z = \frac{1}{c} (E_0 \vec{e}_x - E_0 \vec{e}_y).$$

$$\vec{E}_{\text{ave}} = \frac{1}{2} E_0 (\vec{E} + \vec{H}) = E_0 \frac{1}{2} (\vec{e}_x + \vec{e}_y).$$

Ex. 13 From Gauss' law, $\vec{E} = E \frac{\vec{r}}{r^2}$, where ρ is the free charge density on the inner conductor.

$$V_1 = - \int_0^R \vec{E} \cdot d\vec{r} = - \frac{E}{r} \ln\left(\frac{1}{R}\right) \implies E = E_0 \frac{V_0}{\ln(R/a)}$$

From Ampere's circuital law, $\vec{H} = E_0 \frac{\vec{r}}{2\pi r^2}$.

$$\text{Poynting vector, } \vec{S} = \vec{E} \times \vec{H} = E_0 \frac{V_0 \vec{r}}{2\pi r^2 \ln(R/a)}$$

Power transmitted ϕ per cross-sectional area:

$$\mathcal{P} = \int \vec{S} \cdot d\vec{A} = \frac{V_0^2}{2\pi \ln(R/a)} \int_0^{2\pi} \int_a^R \left(\frac{1}{r^2}\right) r dr d\phi = \frac{V_0^2}{2}.$$

Ex. 14 a) $\vec{E} = \frac{1}{\sqrt{2}} \frac{1}{r^2} \hat{r}$, $\vec{B} = B_0 \hat{z}$.

b) $\vec{E}(x, y) = E_0 \vec{e}_x e^{i\omega t} \cos(k_0 y - \frac{1}{2})$.

$$E_0 = (1 + \frac{1}{2}) \frac{1}{r^2} = (1 + \frac{1}{2}) \frac{1}{r^2} = \frac{3}{2} e^{i\omega t}$$

$$\vec{E}(x, y) = E_0 \frac{1}{r^2} \vec{e}_x e^{i\omega t} \cos(k_0 y - \frac{1}{2}) = \frac{3}{2} \vec{e}_x$$

c) $\vec{E}_{\text{ave}} = \frac{1}{2} E_0 (\vec{E} + \vec{H}) = E_0 \frac{1}{2} \frac{1}{r^2} \vec{e}_x \cos \frac{1}{2}$
 $= E_0 \frac{1}{2} \left(\frac{3}{2} \right) \quad (\text{value}).$

Exercise 1 Given $\vec{E}_1 = E_0 (\vec{e}_1 + j \vec{e}_2) e^{j\omega t}$

1) Assume polarized $\vec{E}_2(t) = (E_0 E_{02} + j E_0 E_{01}) e^{j\omega t}$

Boundary condition at $z=0$: $\vec{E}_1(0) + \vec{E}_2(0) = 0$

$\rightarrow \vec{E}_2(0) = -\vec{E}_1(0) = -j E_0 \vec{e}_2 + E_0 \vec{e}_1 e^{j\omega t}$ as both-charged conductivity
polarized wave is a direction.

2) $E_{02} = (\vec{E}_1 + \vec{E}_2) \cdot \vec{e}_2 \rightarrow -E_0 (\vec{E}_1(0) + \vec{E}_2(0)) \cdot \vec{e}_2 = E_0 (E_{02} + E_0 \cos(\omega t))$

$\vec{E}_1(0) = \frac{1}{2} E_0 + \vec{E}_2(0) = \frac{1}{2} E_0 (\vec{e}_1 + j \vec{e}_2)$, $\vec{E}_2(0) = \frac{1}{2} E_0 - \vec{E}_1(0) = \frac{1}{2} E_0 (-\vec{e}_1 + j \vec{e}_2)$

$\vec{E}_1(0) \cdot \vec{E}_2(0) = E_0 \cos(\omega t) = \frac{1}{4} E_0^2 (\vec{e}_1 + j \vec{e}_2) \cdot (-\vec{e}_1 + j \vec{e}_2)$

$E_0 = -E_0 + \vec{E}_2(0) = \frac{1}{2} E_0 (-\vec{e}_1 + j \vec{e}_2)$

3) $\vec{E}_1(t) = E_0 (\vec{E}_1(0) + \vec{E}_2(0)) e^{j\omega t}$

$= E_0 E_0 \left[(\vec{e}_1 + j \vec{e}_2) e^{j\omega t} + (-\vec{e}_1 + j \vec{e}_2) e^{j\omega t} \right] e^{j\omega t}$

$= E_0 E_0 \left[(-j \vec{e}_1 \vec{e}_2 + j \vec{e}_2 \vec{e}_1) \right] e^{j\omega t}$

$= E_0 E_0 \sin(2\omega t) (\vec{e}_1 \otimes \vec{e}_2 - \vec{e}_2 \otimes \vec{e}_1)$

Exercise 2 Given $\vec{E}_1(x, z) = E_0 \cos(x) e^{j\omega t + jkz}$ (TE₀)

a) $E_0 = 10$, $E_0 \cos(x) = 10 \cos(x)$ as both E_0 and $\cos(x)$ are 10 (max/min)

$E_0 = 10 \cos(x) = 10 \cos(x) = 10 \cos(x)$ as both E_0 and $\cos(x)$ are 10 (max/min)

b) $\vec{E}_1(x, y, z) = E_0 \cos(x) e^{j\omega t + jkz}$ (TE₀)

$\vec{E}_1(x, z) = \frac{1}{2} E_0 \cos(x) \left((\vec{E}_1 + \vec{E}_2) \cdot \vec{e}_1 + (\vec{E}_1 - \vec{E}_2) \cdot \vec{e}_2 \right)$

$= \frac{1}{2} E_0 \cos(x) (E_0 \cos(x) + E_0 \cos(x)) = \frac{1}{2} E_0^2 \cos^2(x) e^{j\omega t + jkz}$

$\vec{E}_1(x, z) = \left(\frac{1}{2} E_0 \cos(x) + \frac{1}{2} E_0 \cos(x) \right) \cos(x) e^{j\omega t + jkz}$ (TE₀)

c) $\cos(x) = \frac{1}{2} E_0 \cos(x) = \left(\frac{1}{2} \right) E_0 = 10 \rightarrow E_0 = 20$ as $\cos(x) = 10$

d) $\vec{E}_1(x, z) = E_0 \cos(x) e^{j\omega t + jkz}$ (TE₀)

$\vec{E}_1(x, z) = \frac{1}{2} E_0 \cos(x) \left((\vec{E}_1 + \vec{E}_2) \cdot \vec{e}_1 + (\vec{E}_1 - \vec{E}_2) \cdot \vec{e}_2 \right)$

$= \frac{1}{2} E_0 \cos(x) (E_0 \cos(x) + E_0 \cos(x))$

e) $\vec{E}_1(x, z) = E_0 \cos(x) + E_0 \cos(x) = E_0 \cos(x) e^{j\omega t + jkz}$

$= E_0 \cos(x) e^{j\omega t + jkz}$ (TE₀)

$\vec{E}_1(x, z) = \vec{E}_1(x, z) + \vec{E}_2(x, z) = (\vec{E}_1 + \vec{E}_2) \cos(x) e^{j\omega t + jkz}$ (TE₀)

Ex. 12.12 (a) From $E_1(x, y) = E_1(x_0 + y_0) e^{i\omega(t - y_0/c)}$ (12.12a),

(b) $E_2 = -\frac{1}{2} E_1$, $E_3 = 0 \implies E = \sqrt{E_1^2 + E_2^2} = \frac{1}{2} E_1$ (12.12b).

$I = \frac{1}{2} \epsilon_0 c E_1^2 = \frac{1}{2} \epsilon_0 c \left(\frac{1}{2} E_1 \right)^2 = \frac{1}{4} \epsilon_0 c E_1^2$ (12.12c)

(c) $E_1(x, y, z) = E_1(x_0 + y_0) e^{i\omega(t - (x_0 + y_0)/c)} = E_1(x_0 + y_0) e^{i\omega(t - x_0/c)}$ (12.12d)

$E_2(x, y, z) = \frac{1}{2} E_1 = \frac{1}{2} E_1 e^{i\omega(t - x_0/c)} = \frac{1}{2} E_1 \left(\frac{1}{2} E_1(x_0 + y_0) \right) e^{i\omega(t - x_0/c)}$
 $= E_1 \left(\frac{1}{4} E_1 \right) e^{i\omega(t - x_0/c)}$

$E_3(x, y, z) = E_1 \left(\frac{1}{4} E_1 \right) e^{i\omega(t - x_0/c)} = \frac{1}{4} E_1^2 e^{i\omega(t - x_0/c)}$ (12.12e)

(d) $\cos E_1 = E_2 = \frac{1}{2} \implies E_1 = \cos^{-1} \left(\frac{1}{2} \right) = 60^\circ$

(e) Consideration $E_1 = E_2 = E_3 = 0$ and $E_1(x, y, z) = E_2(x, y, z) = 0$ and so

$E_3(x, y, z) = E_1(x_0 + y_0) e^{i\omega(t - x_0/c)}$ (12.12f)

$E_3(x, y, z) = \frac{1}{2} E_1 = \frac{1}{2} E_1 e^{i\omega(t - x_0/c)} = \frac{1}{2} E_1 \left(\frac{1}{2} E_1 \right) e^{i\omega(t - x_0/c)} = \frac{1}{4} E_1^2 e^{i\omega(t - x_0/c)}$ (12.12g)

(f) $E_1(x, y, z) = E_2(x, y, z) = E_3(x, y, z) = \frac{1}{2} E_1$ and so $E_1 = \frac{1}{2} E_1$ and so $E_1 = 0$ (12.12h)

$E_2(x, y, z) = E_3(x, y, z) = \frac{1}{2} E_1 = \frac{1}{2} E_1 \left(\frac{1}{2} E_1 \right) = \frac{1}{4} E_1^2$ (12.12i)

Ex. 12.13 (a) From $E_1(x, y, z) = E_1 e^{i\omega(t - y/c)}$ and (12.12a):

$E_2(x, y, z) = E_1 \left(\frac{1}{2} E_1 \right) e^{i\omega(t - y/c)} = \frac{1}{2} E_1^2 e^{i\omega(t - y/c)}$

$E_3(x, y, z) = \frac{1}{2} E_1 \left(-\frac{1}{2} E_1 \right) e^{i\omega(t - y/c)} = -\frac{1}{4} E_1^2 e^{i\omega(t - y/c)}$
 $= \frac{1}{4} E_1^2 e^{i\omega(t - y/c)} e^{i\pi}$

(b) $E_1 = \frac{1}{2} E_2 \left(E = \frac{1}{2} \right) = E_2 \left(\frac{1}{2} E \right) = \frac{1}{2} E \implies E = 0$ (12.12b)

Ex. 12.14 (a) From $E_1(x, y, z) = E_1 e^{i\omega(t - y/c)}$ and (12.12a):

$E_2(x, y, z) = E_1 \left(\frac{1}{2} E_1 \right) e^{i\omega(t - y/c)} = \frac{1}{2} E_1^2 e^{i\omega(t - y/c)}$
 $= \frac{1}{2} E_1^2 e^{i\omega(t - y/c)} e^{i\pi}$

$E_3(x, y, z) = E_1 \left(\frac{1}{2} E_1 \right) e^{i\omega(t - y/c)} = \frac{1}{2} E_1^2 e^{i\omega(t - y/c)}$

(b) $E_1 = \frac{1}{2} E_2 \left(E = \frac{1}{2} \right) = E_2 \left(\frac{1}{2} E \right) = \frac{1}{2} E \implies E = 0$ (12.12b)

Ex. 12.15 For normal incidence: $I = I_0 = \frac{1}{2} \epsilon_0 c E_0^2$, where $|E_0| \leq 1$

$I_0 = \frac{1}{2} \epsilon_0 c E_0^2 = \frac{1}{2} \epsilon_0 c E_1^2 \implies E_1 = \sqrt{\frac{2I_0}{\epsilon_0 c}}$

$\therefore E = \frac{1}{\sqrt{1 + \frac{I_0}{I_0}}} E_1 \implies E_1 = \sqrt{1 + \frac{I_0}{I_0}} E = \sqrt{2} E$ (12.12b)

Ex. 11. a) In the decay reaction (reaction 1):

$$E_1 = E_2 E_3 e^{i\theta_2} e^{i\theta_3} / 2E_1,$$

where $\frac{d\sigma}{d\Omega dE_2 dE_3} = \frac{1}{4} \frac{1}{E_1^2} \left[\sqrt{1 - \frac{E_2^2}{E_1^2}} \right]^2 = \frac{1}{4} \frac{1}{E_1^2} \left[\sqrt{1 - \frac{E_3^2}{E_1^2}} \right]^2$

Given: $\beta_1 = 0$ (rest), $\rightarrow$ $\cos\theta_1 = 1$ or $\theta_1 = 0^\circ$

Let $E_2 = \frac{E_1}{2} = 0.5$ $\rightarrow$ $E_3 = 1.5$ (by eqn. 1), $\beta_2 = 0.6$ or $\cos\theta_2 = 0.6$

$E_2 \sqrt{1 - \beta_2^2} = \frac{(0.5)(1)}{(1.5)(1.5)} e^{i\theta_2} e^{i\theta_3} = 0.111 e^{i\theta_2} e^{i\theta_3}$

$E_1 = E_2 E_3 e^{i\theta_2} e^{i\theta_3} \quad E_1 = E_2 \left(\frac{1}{E_3} \right) = E_2 \frac{1}{1.5} e^{i\theta_2} e^{i\theta_3} = 0.333 e^{i\theta_2} e^{i\theta_3}$

Let $E_1 = E_2 E_3 e^{i\theta_2} \rightarrow E_1 = 0.333 e^{i\theta_2} e^{i\theta_3} \quad E_1 = 0.333 e^{i\theta_2} e^{i\theta_3}$

Equating eqns. $\left\{ \begin{array}{l} 0.111 = E_1 \\ 0.333 = E_1 \end{array} \right. \rightarrow E_1 = 0.111 e^{i\theta_2} e^{i\theta_3} \quad E_1 = 0.333 e^{i\theta_2} e^{i\theta_3}$

$\therefore \frac{E_1}{0.111} = \frac{E_1}{0.333} \rightarrow \cos(\theta_2 + \theta_3) = \cos(\theta_2 + \theta_3) \quad \text{or}$
 $\frac{E_1}{0.111} = \frac{E_1}{0.333} \rightarrow \cos(\theta_2 + \theta_3) = \cos(\theta_2 + \theta_3) \quad \text{or}$
 $\frac{E_1}{0.111} = \frac{E_1}{0.333} \rightarrow \cos(\theta_2 + \theta_3) = \cos(\theta_2 + \theta_3) \quad \text{or}$
 $\frac{E_1}{0.111} = \frac{E_1}{0.333} \rightarrow \cos(\theta_2 + \theta_3) = \cos(\theta_2 + \theta_3) \quad \text{or}$

b) $\cos\theta_2 = \frac{E_2}{E_1} \left(\frac{1}{E_3} - \frac{1}{E_1} \right) = \frac{0.5}{1.5} \left(\frac{1}{1.5} - \frac{1}{0.5} \right)$

$\cos\theta_2 = \frac{0.5}{1.5} \left(\frac{1}{1.5} - \frac{1}{0.5} \right) = \frac{0.5}{1.5} \left(\frac{1}{1.5} - \frac{1}{0.5} \right) = \frac{0.5}{1.5} \left(\frac{1}{1.5} - \frac{1}{0.5} \right)$

Ex. 12 a) $\beta = \frac{v}{c} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \quad E_2 \sqrt{1 - \beta^2} = \frac{E_1}{2} \quad E_2 = \frac{E_1}{2\sqrt{1 - \beta^2}}$

b) $\beta^2 = \left[\frac{E_2}{E_1} \right]^2 = \left[\frac{1 - \frac{E_2}{E_1}}{1 + \frac{E_2}{E_1}} \right]^2 = \left[\frac{1 - \frac{E_2}{E_1}}{1 + \frac{E_2}{E_1}} \right]^2$

Fraction of power absorbed, $F = 1 - \beta^2 = \frac{1}{1 + \frac{E_2}{E_1}}$

c) $\omega = 2\pi \times 10^8 \text{ (Hz)}$ For $\omega = 2\pi \times 10^8 \text{ (Hz)}$ $\beta = \frac{v}{c} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$

$$\text{E.1.11} \quad C = \frac{2 \sin \theta - 2}{2 \sin \theta + 2} = 2 \sin \theta - 2 \frac{2 \sin \theta - 2 \sin \theta + 2}{2 \sin \theta + 2}$$

$$C = \frac{2 \sin \theta - 2}{2 \sin \theta + 2} = \frac{2}{2} - \frac{2 \sin \theta}{2 \sin \theta + 2}$$

$$C = \frac{2 \sin \theta}{2 \sin \theta + 2} = \frac{2}{2} - \frac{2 \sin \theta}{2 \sin \theta + 2}$$

$$\begin{aligned} \therefore E &= \frac{2 \sin \theta \left(\frac{2 \sin \theta - 2}{2 \sin \theta + 2} - \frac{2}{2} \right) + 2 \left(\frac{2}{2} - \frac{2 \sin \theta}{2 \sin \theta + 2} \right)}{2 \sin \theta \left(\frac{2 \sin \theta - 2}{2 \sin \theta + 2} - \frac{2}{2} \right) + 2 \left(\frac{2}{2} - \frac{2 \sin \theta}{2 \sin \theta + 2} \right)} \\ &= \frac{2 \sin \theta \left(\frac{2 \sin \theta - 2 - 2 \sin \theta - 2}{2 \sin \theta + 2} \right) + 2 \left(\frac{2 \sin \theta - 2 \sin \theta + 2}{2 \sin \theta + 2} \right)}{2 \sin \theta \left(\frac{2 \sin \theta - 2 - 2 \sin \theta - 2}{2 \sin \theta + 2} \right) + 2 \left(\frac{2 \sin \theta - 2 \sin \theta + 2}{2 \sin \theta + 2} \right)} \\ &= \frac{2 \sin \theta (-4) + 2(2)}{2 \sin \theta (-4) + 2(2)} \end{aligned}$$

$$\text{E.1.12} \quad E_1 = E_0 (E_0 e^{i\omega t} + E_0 e^{-i\omega t}),$$

$$\bar{E}_1 = E_0 \frac{1}{2} (E_0 e^{i\omega t} + E_0 e^{-i\omega t}),$$

$$E_2 = E_0 (E_1' e^{i\omega t} + E_1' e^{-i\omega t}),$$

$$\bar{E}_2 = E_0 \frac{1}{2} (E_1' e^{i\omega t} + E_1' e^{-i\omega t}).$$

$$\text{At } t = 0, \quad E_2 = 0 \implies E_1' = -E_1' e^{-i\omega t},$$

$$E_1 = E_0 E_1' (e^{i\omega t} + e^{-i\omega t + i\omega t}),$$

$$E_2 = E_0 \frac{1}{2} (E_1' e^{i\omega t} + E_1' e^{i\omega t + i\omega t}).$$

$$\text{Boundary conditions: } E_1(0) = E_2(0) = E_0 = E_0 = E_1' (1 + e^{-i\omega t}),$$

$$E_1(0) = E_2(0) = E_0 = E_0 = E_1' \frac{1}{2} (1 + e^{-i\omega t}).$$

$$E_1' = \frac{2 E_0 E_0}{E_0 (1 + 1) + E_0 (1 + 1) e^{-i\omega t}}$$

$$E_2 = - \left(\frac{2 E_0 - (2 E_0 \sin \theta \cos \theta)}{2 E_0 + 2 E_0 \sin \theta \cos \theta} \right) E_0$$

$$\Rightarrow E_1(x, y) = E_1(x_0 + \cos(\beta)z = \frac{1}{2}(1+x), \quad r = r_0 + r_0 \cos(\beta) \frac{1}{2} \cos(\beta) \frac{1}{2}$$

$$\Rightarrow E_1(x, y) = E_1(x_0 + \cos(\beta)z = \frac{1}{2}(1+x) + \cos(\beta)z = \frac{1}{2}(1+x)$$

$$\Rightarrow E_1(x, y) = E_1 \frac{1 + \cos(\beta)}{2(1 + \cos(\beta) + \cos(\beta) + \cos(\beta))} \left(\cos(\beta)z = \frac{1}{2}(1+x) \right)$$

$$= \cos(\beta)z = \frac{1}{2}(1+x) \frac{1 + \cos(\beta)}{2(1 + \cos(\beta) + \cos(\beta) + \cos(\beta))}$$

$$P = \cos(\beta) \left(\frac{1 + \cos(\beta)}{2(1 + \cos(\beta) + \cos(\beta) + \cos(\beta))} \right)$$

$$\Rightarrow (E_{1,0}) = \frac{1}{2} \cos(\beta) (1 + \cos(\beta)) = 0$$

$$\Rightarrow (E_{1,0}) = 0$$

$$\Rightarrow \cos(\beta) = \cos(\beta) \rightarrow \cos(\beta) = 0 \rightarrow \beta = \pi/2, \cos(\beta) = 0$$

$$E_1(x, y) = E_1 = E_1 - \cos(\beta) = (1 - \cos(\beta)) \frac{1}{2}, \quad r_1 = E_1 = \frac{1}{2} \sqrt{1 + \cos(\beta)}$$

$$r_2 = (1 + \cos(\beta)) \frac{1}{2} = r_2 \quad \text{at } \cos(\beta) = 0$$

$$\Rightarrow \text{From Problem 1.1.1.1}$$

$$E_1' = E_1 \cos(\beta) = \cos(\beta) \frac{1}{2} \frac{1 + \cos(\beta)}{2(1 + \cos(\beta) + \cos(\beta) + \cos(\beta))}$$

$$\Rightarrow E_1' = E_1 \cos(\beta) = \cos(\beta) \frac{1}{2} \frac{1 + \cos(\beta)}{2(1 + \cos(\beta) + \cos(\beta) + \cos(\beta))}$$

$$\Rightarrow E_1' = E_1 \cos(\beta) = \cos(\beta) \frac{1}{2} \frac{1 + \cos(\beta)}{2(1 + \cos(\beta) + \cos(\beta) + \cos(\beta))}$$

$$\Rightarrow E_1' = \frac{1}{2} \frac{1 + \cos(\beta)}{2(1 + \cos(\beta) + \cos(\beta) + \cos(\beta))}$$

$$= \left(1 + \cos(\beta) \right) \frac{1 + \cos(\beta)}{2(1 + \cos(\beta) + \cos(\beta) + \cos(\beta))} E_1$$

$$(E_{1,0}) = \frac{1}{2} \cos(\beta) (E_1 + E_1) = (E_1 + E_1)$$

$$= \frac{1}{2} \cos(\beta) (1 + \cos(\beta))$$

$$\text{where } \frac{1 + \cos(\beta)}{2(1 + \cos(\beta) + \cos(\beta) + \cos(\beta))} = \cos(\beta)$$

$$(E_{1,0}) = \frac{1}{2} \cos(\beta) \frac{1 + \cos(\beta)}{2(1 + \cos(\beta) + \cos(\beta) + \cos(\beta))} = \cos(\beta)$$

$$(E_{1,0}) = \frac{1}{2} \cos(\beta) \frac{1 + \cos(\beta)}{2(1 + \cos(\beta) + \cos(\beta) + \cos(\beta))} = \cos(\beta)$$

$$\begin{aligned} \frac{\partial \hat{A}_z}{\partial x_1} &= \frac{1}{2} \left(\frac{\partial}{\partial x} \right) \frac{1}{\sin \theta_1 \cos \theta_1 \sin \theta_2 \cos \theta_2} \\ \frac{\partial \hat{A}_z}{\partial x_1} &= \frac{1}{2} \frac{\partial}{\partial x} \hat{A}_z \\ \frac{\partial \hat{A}_z}{\partial x_1} &= \frac{1}{2} \left(\frac{\partial}{\partial x} \right) \frac{1}{\sin \theta_1 \cos \theta_1 \sin \theta_2 \cos \theta_2} \\ \text{At } x=0^+ \text{ (air), } \theta_1 = 0, \text{ and } \theta_2 = 0, \quad \hat{A}_z = \hat{A}_z = 0, \quad \frac{\partial \hat{A}_z}{\partial x_1} = 0. \end{aligned}$$

Ex. 14 Given $\beta = \beta_0/\beta_1$ and $\eta = 0$.

$$\rightarrow \gamma_1 = \gamma_0 \sqrt{1 - \beta_0^2} = -\beta_0 \gamma_0, \quad \gamma_0/\gamma_1 = -\beta_0 \gamma_0.$$

$$\begin{aligned} \text{From Eq. (1-20): } \sin \theta_1 &= \frac{\beta_0}{\beta_1} \sin \theta_2 = -\beta_0, \quad \cos \theta_1 = 0, \quad \sin \theta_2 = 0 \\ \text{At } x=0^+ \text{ (air), } \gamma_1 &= \frac{\beta_0 \gamma_0 \cos \theta_1}{\gamma_0 \gamma_1 \cos \theta_2 + \sin \theta_2} = 0. \end{aligned}$$

$$\text{From Eq. (1-21): } \gamma_2 = \frac{\beta_0 \gamma_0 \sin \theta_1}{\gamma_0 \gamma_1 \cos \theta_2 + \sin \theta_2} = 0.$$

$$\text{From Eq. (1-22): } \gamma_3 = \frac{\beta_0 \gamma_0 \sin \theta_1 \cos \theta_2}{\gamma_0 \gamma_1 \cos \theta_2 + \sin \theta_2} = 0.$$

$$\text{From Eq. (1-23): } \gamma_4 = \frac{\beta_0 \gamma_0 \sin \theta_1 \sin \theta_2}{\gamma_0 \gamma_1 \cos \theta_2 + \sin \theta_2} = 0.$$

$\gamma_1 = \gamma_2 = 0$, but the phase shift of the reflected wave depends on the polarization of the incident wave. There are standing waves in the air and exponentially decaying transmitted waves in the hemisphere.

$$\text{Ex. 15} \quad k_{1x}^2 + k_{1y}^2 = k_1^2 = \omega^2 \mu_0 \epsilon_0 = \beta_0^2 k_0^2. \quad (1)$$

Continuity conditions at $x=0$ for all ω and θ_1 :

$$k_{1x} = k_{2x} = k_0 \sqrt{\epsilon_2} \sin \theta_2 = \beta_0 \sqrt{\epsilon_2} k_0. \quad (2)$$

$$k_{1y} = \beta_0 k_0. \quad (3)$$

Combining (1), (2) and (3), we obtain for k_{1x} and β_0 in terms of ω , μ_0 , ϵ_0 , ϵ_2 , and β_0 . But, then

$$J_{\text{tot}}^2 = \frac{1}{2} \omega^2 J_{\text{tot}}^2 \quad \text{or} \quad \omega^2 J_{\text{tot}}^2 = 2 J_{\text{tot}}^2$$

$$\text{we have } J_{\text{tot}}^2 = J_{\text{tot}}^2 = \frac{1}{2} \omega^2 J_{\text{tot}}^2 = \frac{1}{2} \omega^2 J_{\text{tot}}^2 = \frac{1}{2} \omega^2 J_{\text{tot}}^2$$

$$a) \quad J_{\text{tot}}^2 = \frac{1}{2} \omega^2 J_{\text{tot}}^2 = \frac{1}{2} \omega^2 J_{\text{tot}}^2 = \frac{1}{2} \omega^2 J_{\text{tot}}^2 = \frac{1}{2} \omega^2 J_{\text{tot}}^2$$

$$b) \quad J_{\text{tot}}^2 = \frac{1}{2} \omega^2 J_{\text{tot}}^2 = \frac{1}{2} \omega^2 J_{\text{tot}}^2 = \frac{1}{2} \omega^2 J_{\text{tot}}^2 = \frac{1}{2} \omega^2 J_{\text{tot}}^2$$

$$c) \quad J_{\text{tot}}^2 = \frac{1}{2} \omega^2 J_{\text{tot}}^2$$

$$J_{\text{tot}}^2 = \frac{1}{2} \omega^2 J_{\text{tot}}^2 = \frac{1}{2} \omega^2 J_{\text{tot}}^2 = \frac{1}{2} \omega^2 J_{\text{tot}}^2 = \frac{1}{2} \omega^2 J_{\text{tot}}^2$$

$$\frac{J_{\text{tot}}^2}{J_{\text{tot}}^2} = \frac{1}{2} \omega^2 J_{\text{tot}}^2 = \frac{1}{2} \omega^2 J_{\text{tot}}^2 = \frac{1}{2} \omega^2 J_{\text{tot}}^2 = \frac{1}{2} \omega^2 J_{\text{tot}}^2$$

$$d) \quad J_{\text{tot}}^2 = \frac{1}{2} \omega^2 J_{\text{tot}}^2 = \frac{1}{2} \omega^2 J_{\text{tot}}^2 = \frac{1}{2} \omega^2 J_{\text{tot}}^2 = \frac{1}{2} \omega^2 J_{\text{tot}}^2$$

Ex. 11



$$a) \quad J_{\text{tot}}^2 = \frac{1}{2} \omega^2 J_{\text{tot}}^2$$

$$\frac{J_{\text{tot}}^2}{J_{\text{tot}}^2} = \frac{1}{2} \omega^2 J_{\text{tot}}^2$$

$$b) \quad J_{\text{tot}}^2 = \frac{1}{2} \omega^2 J_{\text{tot}}^2$$

$$c) \quad J_{\text{tot}}^2 = \frac{1}{2} \omega^2 J_{\text{tot}}^2$$

$$J_{\text{tot}}^2 = \frac{1}{2} \omega^2 J_{\text{tot}}^2 = \frac{1}{2} \omega^2 J_{\text{tot}}^2 = \frac{1}{2} \omega^2 J_{\text{tot}}^2 = \frac{1}{2} \omega^2 J_{\text{tot}}^2$$

$$d) \quad J_{\text{tot}}^2 = \frac{1}{2} \omega^2 J_{\text{tot}}^2 = \frac{1}{2} \omega^2 J_{\text{tot}}^2 = \frac{1}{2} \omega^2 J_{\text{tot}}^2 = \frac{1}{2} \omega^2 J_{\text{tot}}^2$$

Ex. 12

$$a) \quad J_{\text{tot}}^2 = \frac{1}{2} \omega^2 J_{\text{tot}}^2 = \frac{1}{2} \omega^2 J_{\text{tot}}^2 = \frac{1}{2} \omega^2 J_{\text{tot}}^2 = \frac{1}{2} \omega^2 J_{\text{tot}}^2$$

From Eqs (1-44b) and (1-44c):

$$E_f(x, z) = E_0 E_2 e^{-\gamma_2 z} e^{i k_0 x},$$

$$E_f(x, z) = \frac{E_0}{\sqrt{\epsilon_2}} (E_1 \rho_1 + E_2 \sqrt{\epsilon_2} \cos \theta_1) e^{-\gamma_2 z} e^{i k_0 x},$$

where $\rho_1 = E_1 \cos \theta_1 = E_0 \sqrt{\frac{\epsilon_2}{\epsilon_1}} \sin \theta_1$,

$$\gamma_2 = E_0 \sqrt{\left(\frac{\epsilon_2}{\epsilon_1}\right) \cos^2 \theta_1 - 1}.$$

$$E_2 = \frac{E_0 \cos \theta_1 E_0}{E_0 \cos \theta_1 \sqrt{\left(\frac{\epsilon_2}{\epsilon_1}\right) \cos^2 \theta_1 - 1}} \quad \text{from Eq. (1-44b)}$$

$$d) \quad \partial E_{\text{ref}} / \partial z = \frac{d}{dz} (E_2 e^{-\gamma_2 z}) = 0.$$

Ex. 1.11 Given $\theta_1 = \theta_2 \rightarrow \theta_2 = \pi/2$, $\cos \theta_2 = 0$.

a) From Eq. (1-44b): $(E_{\text{ref}}/E_0)_2 = 2$.

b) From Eq. (1-44b): $(E_{\text{ref}}/E_0)_2 = 2 \epsilon_2 / \epsilon_1$.

$$c) \quad E_f(x, z) = E_0 E_2 \cos \theta_1 \left(1 - \frac{z}{D} \cos \theta_1 \cos \theta_2\right),$$

$$E_f(x, z) = E_0 E_2 e^{-\gamma_2 z} \cos \theta_1 \left(1 - \frac{z}{D} \cos \theta_1\right)$$

$$= E_0 E_2 e^{-\gamma_2 z} \cos \theta_1 \left(1 - \frac{z}{D} \cos \theta_1\right),$$

where $\gamma = \frac{E_0 \sqrt{\left(\frac{\epsilon_2}{\epsilon_1}\right) \cos^2 \theta_1 - 1}}{D} = 0$ when $\theta = \theta_2$.

Ex. 1.12 a) $\theta_1 = \sin^{-1} \sqrt{\epsilon_2 / \epsilon_1} = \sin^{-1} \sqrt{2/3} = 48.19^\circ$

b) $\theta_2 = \sin^{-1} \epsilon_2 = \sin^{-1} \sqrt{\frac{2}{3}} \cos \theta_1 = 48.19^\circ$, $\cos \theta_2 = 0.671$.

$$E_2 = \frac{E_0 \cos \theta_1 \cos \theta_2}{E_0 \cos \theta_1 \cos \theta_2} = e^{i k_0 z} + e^{i k_0 z}$$

$$c) \quad E_2 = \frac{E_0 \cos \theta_1 \cos \theta_2}{E_0 \cos \theta_1 \cos \theta_2} = 0.671 e^{i k_0 z} + 0.671 e^{i k_0 z}$$

d) The transmitted wave is air carrier as $e^{-\gamma_2 z} \cos \theta_2$

where $\gamma_2 = E_0 \sqrt{\left(\frac{\epsilon_2}{\epsilon_1}\right) \cos^2 \theta_1 - 1} = \frac{E_0}{D} (2.41)$

Attenuation is air for each normal length

$$= 2.41 E_0 e^{-\gamma_2 z} = 0.671 \cos \theta_2$$

E.E-32 When the incident light does strike the dielectric surface, $n_1 = n_2 \cos \theta_1$, $\gamma = \sqrt{\frac{2n_1 n_2}{n_1^2 + n_2^2}}$.

$$\frac{\left(\frac{P_{\text{refl}}}{P_{\text{inc}}}\right)}{\left(\frac{P_{\text{refl}}}{P_{\text{inc}}}\right)} = \frac{2}{\gamma} \gamma^2 = \frac{2n_1 n_2}{n_1^2 + n_2^2}$$

Total reflection does inside the prism at both slanting surfaces because

$$\theta_1 = \sin^{-1} n_2 > \theta_2 = \sin^{-1} \left(\frac{1}{n_2}\right) = \theta_{\text{crit}}$$

On exit from the prism, $n_2 = \sqrt{\frac{2n_1 n_2}{n_1^2 + n_2^2}}$.

$$\frac{\left(\frac{P_{\text{refl}}}{P_{\text{inc}}}\right)}{\left(\frac{P_{\text{refl}}}{P_{\text{inc}}}\right)} = \frac{2}{\gamma} \gamma^2 = \frac{2n_1 n_2}{n_1^2 + n_2^2}$$

$$\therefore \frac{\left(\frac{P_{\text{refl}}}{P_{\text{inc}}}\right)}{\left(\frac{P_{\text{refl}}}{P_{\text{inc}}}\right)} = \left[\frac{2n_1 n_2}{n_1^2 + n_2^2}\right]^2 + \left[\frac{2n_1 n_2}{n_1^2 + n_2^2}\right]^2 = 2\gamma^2$$

E.E-33 a) $n_2 \sin \theta_2 = n_1 \sin(\theta_2 + \theta_1) = n_1 \cos \theta_1$
 $= n_1 \sqrt{1 - \sin^2 \theta_1} = n_1 \sqrt{1 - n_2^2/n_1^2} = \sqrt{n_1^2 - n_2^2}$
 also $n_2 = \frac{1}{\gamma} \sqrt{n_1^2 - n_2^2} = \sqrt{n_1^2 - n_2^2}$, ($n_1 > n_2$)

b) $n_1 \sin \theta_1 = \sin \theta_2 = \sqrt{1 - n_2^2/n_1^2} = 2/\gamma \cos \theta_1$
 $n_1 = \sin^{-1} \cos \theta_1 = \sin \theta_1$

E.E-34 $E_p(\theta=0^\circ) = \frac{2n_1}{n_1 + n_2} = \frac{2\sqrt{2}}{\sqrt{2} + 1}$

a) $E_p(\theta=45^\circ) = \gamma_1 = \frac{2n_1 n_2 \cos \theta_1 \cos \theta_2}{(n_1^2 n_2^2 \cos^2 \theta_1 + \cos^2 \theta_2)} = \frac{2\sqrt{2} \cos 45^\circ \cos 45^\circ}{2 \cos^2 45^\circ + \cos^2 45^\circ}$
 $= \frac{2\sqrt{2} \cos 45^\circ}{2 \cos 45^\circ + 1}$

$E_p(\theta=45^\circ) = \gamma_2 = \frac{2n_1 n_2 \cos \theta_1 \cos \theta_2}{(n_1^2 n_2^2 \cos^2 \theta_1 + \cos^2 \theta_2)} = \frac{2 \cos 45^\circ \cos 45^\circ}{2 \cos^2 45^\circ + 1}$

b) $E_p(\theta=60^\circ) = \gamma_1 = \frac{2n_1 n_2 \cos \theta_1 \cos \theta_2}{(n_1^2 n_2^2 \cos^2 \theta_1 + \cos^2 \theta_2)} = \frac{2 \cos 60^\circ \cos 60^\circ}{2 \cos^2 60^\circ + \cos^2 60^\circ}$
 $= \frac{2 \cos 60^\circ \cos 60^\circ}{2 \cos^2 60^\circ + 1}$

$E_p(\theta=60^\circ) = \gamma_2 = \frac{2n_1 n_2 \cos \theta_1 \cos \theta_2}{(n_1^2 n_2^2 \cos^2 \theta_1 + \cos^2 \theta_2)} = \frac{2 \cos 60^\circ \cos 60^\circ}{2 \cos^2 60^\circ + 1}$

Ex-11 a) For perpendicular polarization and $\mu_1 \neq \mu_2$:

$$\sin \theta_{tr} = \frac{1}{\sqrt{1 + \left(\frac{\mu_2}{\mu_1}\right)^2}}$$

Under condition of no-reflection:

$$\begin{aligned} \cos \theta_i &= \sqrt{1 - \frac{\mu_2^2}{\mu_1^2}} \sin \theta_{tr} = \frac{1}{\sqrt{1 + \left(\frac{\mu_2}{\mu_1}\right)^2}} \\ &= \sin \theta_{tr} \implies \theta_i = \theta_{tr} = \pi/2. \end{aligned}$$

b) For parallel polarization and $\mu_1 \neq \mu_2$:

$$\begin{aligned} \sin \theta_{tr} &= \frac{1}{\sqrt{1 + \left(\frac{\mu_2}{\mu_1}\right)^2}} \\ \cos \theta_i &= \sqrt{1 - \frac{\mu_2^2}{\mu_1^2}} \sin \theta_{tr} = \frac{1}{\sqrt{1 + \left(\frac{\mu_2}{\mu_1}\right)^2}} \\ &= \sin \theta_{tr} \implies \theta_i = \theta_{tr} = \pi/2. \end{aligned}$$

Ex-12 a) $\sin \theta_i = \sqrt{\frac{\mu_2}{\mu_1}}$; $\sin \theta_{tr} = \frac{1}{\sqrt{1 + \left(\frac{\mu_2}{\mu_1}\right)^2}}$



$$\implies \cos \theta_{tr} = \sqrt{\frac{\mu_1}{\mu_2}}$$

$$\therefore \sin \theta_i = \cos \theta_{tr} \quad (\theta_i > \theta_{tr})$$

b) Let $\mu_1/\mu_2 = n$.



Ex-13 a) For perpendicular polarization:

$$C_1 = \frac{\mu_2 \cos \theta_i - \mu_1 \cos \theta_{tr}}{\mu_2 \cos \theta_i + \mu_1 \cos \theta_{tr}}$$

$$\sin \theta_i = \sqrt{\frac{\mu_2}{\mu_1}} \sin \theta_{tr}, \quad \cos \theta_i = \sqrt{1 - \frac{\mu_2}{\mu_1}} \cos \theta_{tr}$$

$$C_1 = \frac{\mu_2 \cos \theta_i - \mu_1 \cos \theta_{tr}}{\mu_2 \cos \theta_i + \mu_1 \cos \theta_{tr}}$$

$$C_1 = \frac{\mu_2 \cos \theta_i}{\mu_2 \cos \theta_i + \mu_1 \cos \theta_{tr}} = \frac{\mu_2 \sqrt{\frac{\mu_2}{\mu_1}} \cos \theta_{tr}}{\mu_2 \cos \theta_{tr} + \mu_1 \sqrt{1 - \frac{\mu_2}{\mu_1}} \cos \theta_{tr}}$$

For parallel polarization:

$$r_p = \frac{\frac{E_1 \cos \theta_1}{\mu_1} - \frac{E_2 \cos \theta_2}{\mu_2}}{\frac{E_1 \cos \theta_1}{\mu_1} + \frac{E_2 \cos \theta_2}{\mu_2}} = \frac{E_1 \cos \theta_1}{E_2 \cos \theta_2} = \frac{E_1}{E_2} \frac{\cos \theta_1}{\cos \theta_2}$$

$$r_p = \frac{1}{\frac{\mu_2}{\mu_1} \left(1 - \left(\frac{n_2}{n_1}\right)^2 \sin^2 \theta_1\right) + \cos^2 \theta_1}$$

② $n_2/n_1 = 2.10$, $\sqrt{\mu_2/\mu_1} = 1.1 \rightarrow r_p = 0.97 \frac{E_1}{E_2} = 0.97$



Fig. 1.11



$$\vec{E}_1 \cos \theta_1 = \vec{E}_r \cos \theta_1 + \vec{E}_2 \cos \theta_2 = \cos \theta_1$$

$$\vec{E}_2 = \vec{E}_1 \cos \theta_1 = \vec{E}_1 \cos \theta_1$$

$$\vec{E}_r \cos \theta_1 = \frac{1}{2} \vec{E}_1 + \vec{E}_1 \cos \theta_1$$

$$= \frac{1}{2} (\vec{E}_1 \cos \theta_1 + \vec{E}_1 \sin^2 \theta_1) \quad (\text{Since } \cos^2 \theta_1 = 1 - \sin^2 \theta_1)$$

$$\cos \theta_1 = \frac{1}{2} (1 + \sin^2 \theta_1) \Rightarrow \cos \theta_1 = \frac{1}{2} \left(1 + \frac{\mu_2}{\mu_1} \left(\frac{n_1^2}{n_2^2} - 1 \right) \right) = \frac{1}{2} \left(\frac{\mu_2}{\mu_1} + 1 \right)$$

a) From Eq. (1.11.10)

$$r_p = \frac{\frac{\mu_2}{\mu_1} \cos \theta_1}{\frac{\mu_2}{\mu_1} \cos \theta_1 + \cos \theta_1}$$

$$\text{where } \left(\frac{\mu_2}{\mu_1} \right) = \left(\frac{n_2}{n_1} \right)^2 = (2.10)^2 = 4.41$$

$$\vec{E}_1 \cos \theta_1 = \vec{E}_r \cos \theta_1 + \vec{E}_2 \cos \theta_2 = \cos \theta_1$$

$$\vec{E}_r \cos \theta_1 = \frac{1}{2} \vec{E}_1 + \vec{E}_1 \cos \theta_1 = \frac{1}{2} (\vec{E}_1 \cos \theta_1 + \vec{E}_1 \sin^2 \theta_1) \Rightarrow \vec{E}_r \cos \theta_1 = \frac{1}{2} \vec{E}_1 \sin^2 \theta_1$$

b) From Eq. (1-111) $\sin \theta_1 = \frac{\beta \sin \theta_2}{\sqrt{\epsilon_1 - \beta^2}}$ (complex).

$$\cos \theta_1 = \sqrt{1 - \sin^2 \theta_1} \quad (\text{complex}).$$

The x - and y -components of $\vec{E}_2$ in Eq. (1-110) in part a) have different amplitudes and are out of phase, indicating that it is elliptically polarized.

8-1-47

$$a) \Gamma_1 = \left. \frac{E_1}{E_2} \right|_{z=0} = \frac{E_1 \cos \theta_1}{E_2 \cos \theta_2} = \frac{E_1}{E_2} = \Gamma_1 = \frac{\epsilon_2 \cos \theta_2 - \epsilon_1 \cos \theta_1}{\epsilon_2 \cos \theta_2 + \epsilon_1 \cos \theta_1}.$$

$$\tau_1 = \left. \frac{E_2}{E_1} \right|_{z=0} = \frac{E_2 \cos \theta_2}{E_1 \cos \theta_1} = \tau_1 \left(\frac{\cos \theta_2}{\cos \theta_1} \right) = \frac{2 \epsilon_2 \cos \theta_2}{\epsilon_2 \cos \theta_2 + \epsilon_1 \cos \theta_1}.$$

b) From part a) we have

$$1 + \Gamma_1 = \tau_1$$

This compares with

$$1 + \Gamma_1 = \tau_1 \left(\frac{\cos \theta_2}{\cos \theta_1} \right) \text{ in Eq. (1-111).}$$

Chapter 9

Theory and Application of Transmission Lines

Ex.1

$$P \cdot \vec{E} = \begin{vmatrix} E_x & E_y & E_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x & y & z \end{vmatrix} = E_x \hat{a}_x + E_y \hat{a}_y \longrightarrow \frac{E^2}{2} = 1$$

$$P \cdot \vec{E} = \begin{vmatrix} E_x & E_y & E_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x & y & z \end{vmatrix} = E_x \hat{a}_x + E_y \hat{a}_y \longrightarrow \frac{E^2}{2} = 1$$

Ex.2

a) $\vec{E} = (E_x \hat{a}_x + E_y \hat{a}_y) = f(y) \hat{a}_x (E_x \hat{a}_x + E_y \hat{a}_y)$

$$\longrightarrow \begin{cases} \frac{\partial E_x}{\partial y} = -\frac{1}{y^2} E_x & 0 \\ \frac{\partial E_y}{\partial y} = -\frac{1}{y^2} E_y & 0 \\ \frac{\partial E_z}{\partial y} = \frac{1}{y^2} & 0 \end{cases}$$

or $(E_x \hat{a}_x + E_y \hat{a}_y) = f(y) (E_x \hat{a}_x + E_y \hat{a}_y)$

$$\longrightarrow \begin{cases} \frac{\partial E_x}{\partial y} = -\frac{1}{y^2} E_x & 0 \\ \frac{\partial E_y}{\partial y} = -\frac{1}{y^2} E_y & 0 \\ \frac{\partial E_z}{\partial y} = \frac{1}{y^2} & 0 \end{cases}$$

From 0 and 0: $\frac{\partial E_x}{\partial y} = -\frac{1}{y^2} E_x$ 0

From 0 or 0: $\frac{\partial E_y}{\partial y} = -\frac{1}{y^2} E_y$ 0

From 0 or 0: $\frac{\partial E_z}{\partial y} = \frac{1}{y^2}$ 0

1) From 0: $\frac{\partial E_x}{\partial y} = -\frac{1}{y^2} E_x$ 0

From 0, 0, and 0: $\frac{\partial E_x}{\partial y} = -\frac{1}{y^2} E_x \longrightarrow \frac{\partial E_x}{\partial y} = -\frac{1}{y^2} E_x$ 0

Combining 0 and 0, we have $\frac{\partial E_x}{\partial y} = -\frac{1}{y^2} E_x$ 0

Similarly, $\frac{\partial E_y}{\partial y} = -\frac{1}{y^2} E_y$ 0

Ex.3 $E_y(r=0) = E_0 = \frac{E_0}{\sqrt{2}}$

a) $E_x = \frac{E_0}{\sqrt{2}} \frac{r}{R} = \frac{E_0}{\sqrt{2}} \frac{r}{R} \longrightarrow \vec{E} = E \hat{a}_r$

b) $E_x = \frac{E_0}{\sqrt{2}} \frac{r}{R} = \frac{E_0}{\sqrt{2}} \frac{r}{R} \longrightarrow \vec{E} = E \hat{a}_r$

$$d) \quad \alpha_n = \frac{1}{n} \sqrt{\frac{n}{n-1}} = \frac{1}{n} \sqrt{\frac{n}{n-1}} \longrightarrow 0 \text{ as } n \rightarrow \infty.$$

$$e) \quad \alpha_p = \frac{1}{p} \sqrt{\frac{p}{p-1}} \longrightarrow \frac{\alpha_{p+1}}{\alpha_p} = \frac{\alpha_p}{\alpha_p} \text{ for some } p, \\ \frac{\alpha_{p+1}}{\alpha_p} = \frac{\alpha_p}{\alpha_p} \text{ for some } p, \\ \alpha_{p+1} = \alpha_p \text{ for some } p.$$

Ex. 2 Given: $x_1 = (1+i)^n$ (real), $x_2 = (1-i)^n$ (real), $x_3 = (1+i)^n(1-i)^n$
 Using addition and subtraction, $y = x_1 + x_2$, $z = x_1 - x_2$,
 $f = 1 + i^n$ (real).

$$a) \quad x = \frac{1}{n} \sqrt{\frac{n}{n-1}} = 1/n \text{ (real)}$$

$$1 + i^n \sqrt{\frac{n}{n-1}} = 1 + i^n \sqrt{\frac{n}{n-1}}$$

$$z = x \sqrt{\frac{n}{n-1}} = 1/n \sqrt{\frac{n}{n-1}}$$

$$z = x \sqrt{\frac{n}{n-1}} = 1/n \sqrt{\frac{n}{n-1}}$$

$$b) \quad \frac{f(x)}{f(y)} = \sqrt{\frac{n}{n-1}} = 1/n \sqrt{\frac{n}{n-1}}.$$

$$c) \quad x_1 = (1+i)^n, \quad x_2 = (1-i)^n$$

$$y = (1+i)^n + (1-i)^n = (1+i)^n + (1-i)^n$$

$$z = (1+i)^n - (1-i)^n = (1+i)^n - (1-i)^n$$

Ex. 3 Addition and subtraction of (1) and (2)

$$(1) + (2) = x_1^2 + x_2^2 = x_1^2 + x_2^2$$

$$(1) - (2) = x_1^2 - x_2^2 = x_1^2 - x_2^2$$

Let $x_1 = (1+i)^n$, $x_2 = (1-i)^n$

$$(1+i)^n + (1-i)^n = (1+i)^n + (1-i)^n$$

which is equal to

$$(1+i)^n + (1-i)^n = 2$$

$$(1+i)^n - (1-i)^n = 2i$$

$$\therefore \frac{x_1^2}{x_2^2} = \frac{x_1^2}{x_2^2} = \frac{1+i}{1-i}$$

Ex. 4 $x = (1+i)^n$, $y = (1-i)^n$, $z = (1+i)^n(1-i)^n$

From Eq. (1) and (2), $y = (1-i)^n = (1-i)^n$

Given: $Z_{in} = 50 + j20 \text{ } \Omega$,
 β is not stated, assume typical,
 $\beta = 100$ (assumed),
 $r_e = 26 \text{ mV}$ (assumed).

$r_e = r_{e1} = 26 \text{ mV} / I_{E1}(\text{mA})$, $I_E = \frac{I_{CQ}}{\beta} = 0.24 \text{ (mA)}$,
 $r_e = \frac{26}{0.24} = 108 \text{ } \Omega$ (calculated), $Z_e = \frac{Z_{in}}{\beta} = 70 \text{ } \Omega$ (calculated).

Ex. 9.10 a) For approximate transmission line:

$Z_L = \sqrt{\frac{L}{C}} = \frac{1}{\beta} \sqrt{\frac{L}{C}} \text{ (calculated)} \left(\frac{Z_{in}}{\beta} \right) = \frac{100}{100} \left(\frac{50 + j20}{100} \right) = 0.5 + j0.2$
 $\frac{L}{C} = 25 \text{ nH} \longrightarrow L = 25.5 \times 10^{-9} \text{ (calculated)}$

b) For exact transmission line:

$Z_L = \frac{1}{\beta} \sqrt{\frac{L}{C}} \left(\beta \left(\frac{Z_{in}}{\beta} \right) \right) = \frac{100}{100} \left(\frac{50 + j20}{100} \right) = 0.5$
 $\frac{L}{C} = 25 \text{ nH} \longrightarrow L = 25.5 \times 10^{-9} \text{ (calculated)}$

Ex. 9.11 $(P_{out})_1 = (P_{out})_2 = \frac{1}{2} R_L |i_L|^2$ $V_1 = \frac{R_1}{R_1 + R_2} V_s$
 $= \frac{(1 \text{ mV})^2 R_L}{(R_1 + R_2)^2 + (R_1 + R_2)^2}$ $V_2 = \frac{R_2}{R_1 + R_2} V_s$

To maximize $(P_{out})_1$, set $\frac{\partial (P_{out})_1}{\partial R_L} = 0$, $\left\{ \begin{array}{l} R_1 = R_2, R_1 = R_2 \\ \text{and } \frac{\partial (P_{out})_1}{\partial R_1} = 0 \end{array} \right\} = \frac{R_1}{R_1 + R_2}$

$\text{Max. } (P_{out})_1 = \frac{V_s^2}{8 R_1} = (P_{out})_2$

$\longrightarrow \text{Max. power transfer efficiency} = 87.5\%$

Ex. 9.12 $V_{TH} = V_s' e^{j\omega t} + V_s'' e^{j\omega t}$,

$I_{TH} = I_s' e^{j\omega t} + I_s'' e^{j\omega t}$

At $\omega = 0$: $V_{TH} = V_s' + V_s''$; $I_{TH} = I_s' + I_s'' = \frac{1}{2} (V_s' - V_s'')$
 $\longrightarrow V_s' = \frac{1}{2} (V_s + I_s R_s)$, $V_s'' = \frac{1}{2} (V_s - I_s R_s)$

a) $V_{TH} = \frac{1}{2} (V_s + I_s R_s) e^{j\omega t} + \frac{1}{2} (V_s - I_s R_s) e^{j\omega t}$

$I_{TH} = \frac{1}{2} (I_s + I_s) e^{j\omega t} - \frac{1}{2} (I_s - I_s) e^{j\omega t}$

b) $V_{TH} = V_s' \cos \omega t + V_s'' \cos \omega t$,

$I_{TH} = I_s' \cos \omega t + I_s'' \cos \omega t$

$$\begin{aligned} \text{Ex-22} \quad \text{From Eq. (1) we have } y_1 &= \frac{1}{2}x_1 + \frac{1}{2}(x_1 - x_2) \\ &= \left(\frac{1}{2} + \frac{1}{2}\right)x_1 - \frac{1}{2}x_2. \end{aligned} \quad (3)$$

$$\begin{aligned} \text{Also, } x_1 &= x_2 + (y_1 - \frac{1}{2}x_1) \\ &= \frac{1}{2}x_2 + \left(1 + \frac{1}{2}\right)x_1. \end{aligned} \quad (4)$$

$$\begin{aligned} \text{Substituting (3) in (4):} \\ x_1 &= \left(1 + \frac{1}{2}\right)x_1 + \frac{1}{2}\left(1 + \frac{1}{2}\right)x_2. \end{aligned} \quad (5)$$

(5) is wrong. If $x_1 = x_2$ and $x_1 x_2 = y_1$ in Eq. (1) then

$$y_1 = y_2 = (\cos 30^\circ)x_1 + (\sin 30^\circ)x_2. \quad (6)$$

$$x_1 = \cos 30^\circ = \left(\frac{1}{2}\right)(\cos 30^\circ)x_1 + (\sin 30^\circ)x_2. \quad (7)$$

$$\begin{aligned} \text{Both Eqs. (6) &\& (7) and Eqs. (3) &\& (4) are of the} \\ \text{following form:} \quad \begin{bmatrix} x \\ y \end{bmatrix} &= \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}. \end{aligned} \quad (8)$$

$$\text{where } a = 1 + \frac{1}{2} = \cos 30^\circ, \quad (9)$$

$$b = \frac{1}{2}\left(1 + \frac{1}{2}\right) = \sin 30^\circ, \quad (10)$$

$$\text{and } c = \frac{1}{2} = \frac{1}{2} \sin 30^\circ. \quad (11)$$

$$\text{--- } d = a = \cos 30^\circ = \cos 30^\circ = c.$$

(8) is the required result in Eq. (1) and
Eq. (2) can be obtained by using (8) in (2):
 $x_1 = \frac{1}{2}(x_1 - y_1) = \frac{1}{2} \cos 30^\circ$

$$\text{Ex-23 } a_1 = \frac{1}{\sqrt{2}} = a_2, \quad -\frac{1}{\sqrt{2}} = a_3.$$

$$\begin{cases} \frac{1}{\sqrt{2}} = a_4, \\ \frac{1}{\sqrt{2}} = a_5. \end{cases}$$

$$\text{a) } y_1(x) = \frac{1}{\sqrt{2}} e^{i\pi x} = \frac{1}{\sqrt{2}} e^{i\pi x}$$

$$y_2(x) = \frac{1}{\sqrt{2}} e^{i\pi x} = \frac{1}{\sqrt{2}} e^{i\pi x}, \quad \dots = \frac{1}{\sqrt{2}}$$

$$\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$$\text{We have } V(t) = \frac{1}{2}(v_1 + v_2)e^{i\omega t} + \frac{1}{2}(v_1 - v_2)e^{-i\omega t}$$

$$I(t) = \frac{1}{2}\left(\frac{d}{dt}v_1 + v_2\right)e^{i\omega t} - \frac{1}{2}\left(\frac{d}{dt}v_1 - v_2\right)e^{-i\omega t}$$

$$\text{where } v_1 = \frac{B_1}{B_2 - B_1}v_2 \quad \text{and } v_2 = \frac{B_2}{B_2 - B_1}v_1$$

a) For an infinite line, $B_1 = B_2$:

$$V(t) = \frac{B_2}{B_2 - B_1}v_2 e^{i\omega t}, \quad I(t) = \frac{B_2}{B_2 - B_1}v_2 e^{i\omega t}$$

d) For a finite line of length L terminated in B_2 :

$$B_1 = B_2 \frac{B_2 + iB_2 \tan \beta L}{B_2 - iB_2 \tan \beta L}$$

Ex. 11 Characteristic line: $B_1 = \sqrt{\frac{\mu}{\epsilon}} = 50 \, \Omega$, $Z = 50 \, \Omega$ (line)

$$\Gamma_{in} \left(\frac{B_2}{B_1} \right) = \Gamma_{in} \left(\frac{B_2}{B_1} \right) = 0 \text{ at } z = 0$$

$$\implies \frac{B_2}{B_1} = 0.5 \text{ at } z = 0, \quad \frac{B_2}{B_1} = 0.5 \text{ at } z = 0.5 \text{ m} = 0.5 \text{ m} = \frac{L}{2}$$

$$L = \frac{B_2}{B_1} = 0.5 \text{ m} = 0.5 \text{ m}, \quad C = \frac{B_2}{B_1} = 0.5 \text{ m} = 0.5 \text{ m}$$

$$Z = \frac{B_2}{B_1} = 0.5 \text{ m} = 0.5 \text{ m}, \quad \beta = 0.5 \text{ m} = 0.5 \text{ m}$$

$$\text{a) } V(t) = \frac{B_2}{B_1} e^{i\omega t} e^{i\beta z} = \frac{B_2}{B_1} e^{i\omega t} e^{i\beta z}, \quad I(t) = \frac{B_2}{B_1} e^{i\omega t} e^{i\beta z}$$

$$\therefore P_{avg} = 0.5 \text{ W} e^{i\omega t} \text{ at } z = 0.5 \text{ m} = 0.5 \text{ W} \quad (1)$$

$$I_{avg} = 0.5 \text{ W} e^{i\omega t} \text{ at } z = 0.5 \text{ m} = 0.5 \text{ W} \quad (2)$$

$$\text{b) } Z = 50 \, \Omega \text{ at } z = 0.5 \text{ m} \implies \Gamma_{in} = 0.5 \text{ at } z = 0.5 \text{ m} = 0.5 \text{ m} \quad (3)$$

$$I_{avg} = 0.5 \text{ W} \text{ at } z = 0.5 \text{ m} = 0.5 \text{ W} \quad (4)$$

$$\text{c) } \Gamma_{in} = \frac{1}{2} \left(\frac{B_2}{B_1} \right) = \frac{1}{2} (0.5 \text{ at } z = 0.5 \text{ m}) = 0.5 \text{ m} \quad (5)$$

Ex. 12 a) From Eq. (9-113) $Z_L = Z_0 \tanh \beta L = Z_0 \tanh \beta L$

From Eqs. (9-113) and (9-113) $\beta = \frac{2\pi}{\lambda} \sqrt{\epsilon_r - 1} = \frac{2\pi}{\lambda} \sqrt{4 - 1} = \frac{2\pi}{\lambda} \sqrt{3}$

$$\therefore Z_L = (1 + j\sqrt{3})Z_0$$

$$\text{b) From Eq. (9-113) } Z_L = Z_0 \tanh \beta L = \frac{Z_0}{2} = \frac{50 \, \Omega}{2} = 25 \, \Omega$$

Ex. 2.12 a) From Eq. (9-100) $Z_{in} = Z_0 \tanh \gamma l = Z_0 \left(\frac{1 - e^{-2\gamma l}}{1 + e^{-2\gamma l}} \right)$.

For $Z_0 = 50 \Omega$, $\beta l = 0.5$, $\alpha = 1$,

$$Z_{in} = 50 \left(\frac{1 - e^{-\frac{2\pi \times 10^8 \times 0.5}{3 \times 10^8}}}{1 + e^{-\frac{2\pi \times 10^8 \times 0.5}{3 \times 10^8}}} \right) = 50 \left(\frac{1 - (0.92388)}{1 + (0.92388)} \right) \\ = 4.29 \Omega.$$

b) From Eq. (9-100) $Z_{in} = Z_0 \coth \gamma l = Z_0 \left(\frac{1 + e^{-2\gamma l}}{1 - e^{-2\gamma l}} \right)$.

For $Z_0 = 50 \Omega$, $Z_{in} = 50 \left(\frac{1 + e^{-\frac{2\pi \times 10^8 \times 0.5}{3 \times 10^8}}}{1 - e^{-\frac{2\pi \times 10^8 \times 0.5}{3 \times 10^8}}} \right) = 50 \left(\frac{1 + (0.92388)}{1 - (0.92388)} \right) \\ = 542.9 \Omega.$

Ex. 2.13 $\beta l = \frac{2\pi f}{v} l = \frac{2\pi}{\lambda} l = 0.5$

$\tan \beta l = \tan 0.5 = -0.5463$,

$$Z_L = Z_0 \frac{Z_0 + j Z_0 \tan \beta l}{Z_0 \tan \beta l} = j Z_0 \frac{(1 + \tan^2 \beta l)}{2 \tan \beta l} = j 50 \frac{(1 + 0.2984)}{2(-0.5463)} \\ = -23.1 \Omega \quad (25^\circ)$$

Ex. 2.14 a) From $Z_L = Z_0 \tanh \gamma l = 200 \Omega$ (25),

$Z_0 = \frac{Z_L}{\tanh \gamma l} = 100 \Omega$ (26).

b) $Z_0 = \sqrt{\frac{L}{C}} Z_L = 100 \Omega$ (27) $= 100 \times 10^{-3} \sqrt{L/C}$ (28).

$\tanh \gamma l = \sqrt{\frac{L}{C}} = 0.5$ (29) $= 0.5175$ (29a) $= \tanh 0.5$ (29b).

$l = 0.5$ (30) $\implies \omega = 2\pi \times 10^8$ (29a),

$\beta = 0.517$ (29b),

c) $Z_1 = \sqrt{\frac{L}{C} \frac{Z_0}{2 - Z_0}} = 75 = \sqrt{\frac{L}{C} \frac{100}{2 - 100}}$,

$\implies L/C = 75^2 \times \frac{2 - 100}{100} = \frac{1}{4}$.

$\omega = \beta v = 0.517 \times 3 \times 10^8 = 1.55 \times 10^8$ (29a).

Wk. values: $L = 0.8 \mu\text{H}$ (31), $C = 0.01 \mu\text{F}$ (32a),

$\omega = 0.314$ (32b), $C = 0.1 \mu\text{F}$ (32c).

P. 9.22 (a) Since the line is very short compared to a wave length, we may use Eqs. (9-1) and (9-2)

$$\left. \begin{aligned} Z &= \frac{Z_0 \tanh \beta}{1 + j Z_0 \tanh \beta} = Z_0 \tanh \beta \quad (\text{since } \beta \ll 1) \\ Y &= \frac{j B \tanh \beta}{1 + j Z_0 \tanh \beta} = j B \tanh \beta \quad (\text{since } \beta \ll 1) \end{aligned} \right\} Z_0 = \sqrt{\frac{L}{C}} = 74.7 \, \Omega$$

$$\beta L = 1.2 \implies L = \frac{\beta L}{\beta} = \frac{1.2}{0.01} = 120 \text{ m.}$$

(b) $\beta = \frac{\omega}{v} = 2\pi \times 10^6 \sqrt{\frac{L}{C}} = 2.41 \text{ (rad/m)}$; $\beta L = 0.41 = 24^\circ$
 $\therefore Z_{in} = Z_0 \tanh \beta L = \frac{1}{j 2.5} = -j 0.4 \, \Omega$
 $Y_{in} = Y_0 \tanh \beta L = 0.025 = 2.5 \text{ mS}$

P. 9.23 From Eq. (9-1) $Z_{in} = Z_0 \tanh \beta L = Z_0 \frac{1 - e^{-2\gamma L}}{1 + e^{-2\gamma L}}$
 $= Z_0 \frac{e^{\gamma L} - e^{-\gamma L}}{e^{\gamma L} + e^{-\gamma L}}$ (9)

For a lossless line, $\gamma = j\beta$; $Z_{in} = Z_0 \frac{e^{j\beta L} - e^{-j\beta L}}{e^{j\beta L} + e^{-j\beta L}}$ (10)

At $L = 0$, $Z_{in} = Z_0$; $(Z_{in})_{\text{max}} = \frac{Z_0}{1 - \cos 2\beta L}$ (11)

When the frequency is slightly off resonance,

$$\beta = \beta_0 + \Delta\beta \quad (\Delta\beta \ll \beta_0); \quad \beta L = \pi \left(1 + \frac{\Delta\beta}{\beta_0} \right); \quad \cos 2\beta L = -1$$

$$\text{and } \sin 2\beta L \approx \sin 2\pi \left(1 + \frac{\Delta\beta}{\beta_0} \right) = 2\pi \Delta\beta \beta_0 L$$

(12) becomes, after leaving out second-order small terms

$$Z_{in} = \frac{Z_0}{1 - j 2\pi \Delta\beta \beta_0 L} \quad (13)$$

Combining (10) & (13) $\frac{Z_{in}}{Z_0} = \frac{1}{1 - j 2\pi \Delta\beta \beta_0 L}$ (14)

Let the power ratio at ω be $\left(\frac{Z_{in}}{Z_0} \right)^2 = 0.1$; so $\Delta\beta = \pm \frac{\sqrt{0.9}}{2\pi \beta_0 L}$ (15)

For $L = 100$ ft, $\beta_0 = \frac{\pi}{16}$ rad, and $\Delta\beta = \pm \frac{\sqrt{0.9}}{32\pi}$

which, for a lossless transmission line, becomes (16)

$$\Delta\beta = \pm \frac{\pi}{32} \left(\frac{1}{L} - \frac{\beta}{L} \right) \implies \frac{\text{half-power frequencies}}{\text{resonant freq.}} = \frac{\beta}{\beta_0} \left(1 \pm \frac{\sqrt{0.9}}{32} \right)$$

$$B = \frac{1}{\sqrt{0.9}} = \frac{1}{\left[\frac{1}{32} \left(\frac{1}{L} - \frac{\beta}{L} \right) \right]^2}$$

Ex. 1.11 (i) For a lossless quarter-wave line, we have

$$\begin{aligned} Z_1 &= \frac{Z_0^2}{Z_2} = \frac{Z_0^2}{Z_1 + jZ_0} = \frac{Z_0^2 Z_1}{Z_0^2 + jZ_0 Z_1} = \frac{Z_0^2 Z_1}{Z_0^2 + jZ_0 Z_1} = Z_1' + jZ_1'' \\ \longrightarrow Z_1 &= \frac{Z_0^2 Z_1}{Z_0^2 + jZ_0 Z_1} \quad \text{--- (1)} \quad Z_1' = \frac{Z_0^2 Z_1}{Z_0^2 + jZ_0 Z_1} \quad \text{--- (2)} \\ &\quad \text{[Resistive } Z_1' \text{ and inductive reactance } Z_1'' \text{ in series]} \end{aligned}$$

Input impedance Z_1 can also be expressed in terms of a resistance R_1 and a capacitive reactance X_1 in parallel:

$$Z_1 = \frac{R_1 X_1}{R_1 + jX_1} = \frac{R_1 X_1}{R_1 + jX_1} = \frac{R_1 X_1}{R_1 + jX_1} = Z_1' + jZ_1'' \quad \text{--- (3)}$$

Combining Eqs. (1), (2), and (3), we find

$$R_1 = \frac{Z_0^2}{Z_1} \quad \text{and} \quad X_1 = -\frac{Z_0^2}{Z_1}.$$

Both of which are reciprocal of Z_1 (E=0).

At $Z_0 = Z_1$ (E=0): $\text{input } Z_1 (Z_1 \text{ input}) = Z_0 \text{ (input)}$.

At the input, $Z_1' = R_1$, $Z_1'' = X_1$, we have

$$Z_1 = R_1 + jX_1 = Z_0 + j0.$$

At the load, $Z_1' = R_1$, $Z_1'' = X_1$, and $Z_1 = R_1 + jX_1 = Z_0 + j0$.

$$\therefore \frac{R_1}{Z_0} = \frac{X_1}{Z_0} = \frac{Z_0}{Z_1 + j0}$$

$$\text{Ex. 1.12 (i) } \Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \left| \frac{Z_L - Z_0}{Z_L + Z_0} \right| = \frac{\sqrt{(R_L - Z_0)^2 + (X_L)^2}}{\sqrt{(R_L + Z_0)^2 + (X_L)^2}},$$

where $Z_L = R_L + jX_L$ and $Z_0 = R_0 + jX_0$.

$$\longrightarrow \Gamma = \frac{1}{2} \left[\frac{R_L + jX_L + R_0 + jX_0}{R_L + jX_L - R_0 - jX_0} \right]^2$$

$$\text{When } R_L = R_0, \quad \Gamma = \frac{1}{2} \sqrt{\frac{X_L + X_0}{X_L - X_0}}.$$

$$\text{At } R_L = R_0 \text{ and } X_L = X_0 = 0 \longrightarrow \Gamma = \frac{1}{2} \sqrt{\frac{0}{0}}.$$

$$Z_L = Z_0 = 0 \text{ (E=0)}$$

4) From Eq. (2-107), $V = j\omega L_e = \frac{j\omega L_0}{1 + \beta^2 L_0^2}$

where $L_0 = L_e / \beta$, and $\beta = \tan \beta L_0$

— $L_0 = \frac{(1 + \beta^2 L_0^2) \frac{j\omega L_0}{1 + \beta^2 L_0^2}}{j\omega} = L_0$

$\beta = 1$ or $\frac{\pi}{4}$ for $L_e = L_0$ and $L_e' = 0$.

Also, $L_e = \frac{j\omega L_0}{1 + \beta^2 L_0^2}$ — $\beta = \frac{1}{\beta L_0} \left[(1 - \beta^2 L_0^2) \pm \sqrt{1 - \beta^2 L_0^2} + \beta^2 L_0^2 \right]$

$L_e = 0$ (able impedance) + (shorted).

For $L_e = \frac{\pi}{4}$, $\beta = \frac{1}{\beta L_0} \left[\frac{1}{\beta L_0} \right] \rightarrow L_0 = 0.5 \text{ mH}$

For $L_e = \frac{\pi}{4}$, $\beta = \frac{1}{\beta L_0} \rightarrow L_0 = 0.5 \text{ mH}$

Use L_0 and β to obtain L_e nearest to the load
at $L_e = 0.5 \text{ mH}$ — shorted.

Ex. 2.11 a) $|r|^2 = \left| \frac{(Z_L + Z_0) + jZ_0}{(Z_L + Z_0) + jZ_0} \right| = \frac{(Z_L + Z_0)^2 + Z_0^2}{(Z_L + Z_0)^2 + Z_0^2}$

$\frac{Z_0^2}{Z_0^2} = 1 \rightarrow Z_0 = \sqrt{Z_L^2 + Z_0^2}$

Let $Z_L = 40 + j10 \text{ } \Omega$, $Z_0 = 50 \text{ } \Omega$

a) $\text{Min } |r| = \sqrt{\frac{Z_L^2 + Z_0^2}{Z_L^2 + Z_0^2}} = \sqrt{\frac{40^2 + 10^2}{50^2}} = \frac{1}{2}$

$\text{Min } \beta = \frac{1 - \frac{1}{2}}{1 + \frac{1}{2}} = \frac{1}{3}$

b) From Eq. (2-107) $V = j\omega L_e = \frac{j\omega L_0}{1 + \beta^2 L_0^2} = 2.0 - j1.0 \text{ } \Omega$

— $\beta = \frac{1}{\beta L_0} \left[(1 - \beta^2 L_0^2) \pm \sqrt{1 - \beta^2 L_0^2} + \beta^2 L_0^2 \right] \left(\frac{2.0 - j1.0}{L_0} \right)$

At voltage minimum, $L_e = \frac{\pi}{4} = \frac{\pi}{4}$

$\beta = 1$ (the negative sign)

$\tan \beta L_0 = \tan \left(\frac{\pi}{4} \right) = 1 \rightarrow L_0 = \frac{\pi}{4}$

$\therefore$ Voltage minimum nearest to the load is $\left(\frac{\pi}{4} - \frac{\pi}{4} \right)$

or 0.25λ from the load.

Ex. 12 a) From $E_p(1-\cos\theta)$ and $C(124)$:

$$V(\theta) = \frac{1}{2}(E_1 + E_2)e^{i\theta} [1 + \gamma e^{i2\theta} e^{i\phi}],$$

$$\text{where } E = \frac{E_1 E_2}{E_1 + E_2} = \gamma^2 e^{i\phi}, \quad \phi = \phi_1 - \phi_2$$

$$\text{then } |V(\theta)| = \left| \frac{1}{2}(E_1 + E_2)e^{i\theta} [1 + \gamma e^{i2\theta} e^{i\phi}] \right| \text{ for } \phi = 0,$$

$$\text{and } |V(\theta)| = \left| \frac{1}{2}(E_1 + E_2)e^{i\theta} [1 + \gamma e^{i2\theta} e^{i\phi}] \right| \text{ for } \phi = \pi.$$

$$S(\theta) = \frac{d|V(\theta)|}{d\theta} = \frac{1}{2}(E_1 + E_2)e^{i\theta} \cdot \left\{ \frac{d|V(\theta)|}{d\theta} \text{ at } \phi = 0 \text{ or } \pi \right\}$$

b) From $E_p(1-\cos\theta)$: $E_1(\theta) = \frac{E_1 E_2}{E_1 + E_2} \frac{1 + \cos\theta}{1 - \cos\theta} E_2$

if a voltage ratio, $\phi = 0$, $E_1(\theta) = S(\theta) E_2$.

c) if a voltage ratio, $\phi = \pi$, $E_1(\theta) = \frac{E_2}{E_1}$.

Ex. 13 From $E_p(1-\cos\theta)$: $E_1 = E_2 \frac{1 + \cos\theta}{1 - \cos\theta} \longrightarrow E_1 = E_2 \frac{1 + \cos\theta}{1 - \cos\theta} = 1 + \cos\theta$

$$E_1 = E_2 \frac{1 + \cos\theta}{1 - \cos\theta} \longrightarrow E_1 = E_2 \frac{1 + \cos\theta}{1 - \cos\theta}.$$

Now $E_1 = 12 \text{ (V)}$ and $E_2 = 40 \text{ (V)}$ and then

$$40 + \gamma = E_2 \frac{1 + \cos\theta}{1 - \cos\theta} \longrightarrow \begin{cases} 40\gamma + 12 = 40E_1 \\ 40\gamma + 12 = -40E_1 \end{cases}$$

$$\therefore E_1 = 16.3 \text{ (V)}, \quad \theta = 180^\circ \text{ or } 0^\circ \text{ or } \pi \longrightarrow E = 16.3 \text{ (V)}.$$

Ex. 12 a) $1 = \frac{1}{2} + \frac{1}{2} = \frac{1}{2} + \frac{1}{2}$

$$E_p(1-\cos\theta) \quad V(\theta) = \frac{1}{2}(E_1 + E_2)e^{i\theta} [1 + \gamma e^{i2\theta}].$$

$$E_p(1-\cos\theta) \quad E = \frac{E_1 E_2}{E_1 + E_2} = \gamma^2 e^{i\phi}, \quad \phi = \phi_1 - \phi_2$$

For $\theta = 0$ is maximum when $\phi = 0 \longrightarrow E_1 = E_2 \frac{1 + \cos\theta}{1 - \cos\theta}$
 $\therefore E = \frac{1}{2} e^{i2\theta}$

b) $E_1 = E_2 \left(\frac{1 + \cos\theta}{1 - \cos\theta} \right) = 40 + 12 \cos\theta \text{ (V)}$

c) Terminating resistance $E_1 = \frac{1}{2} = \frac{40}{2} = 20 \text{ (V)}$

$$E_1 = \frac{1}{2} \gamma E_2 = 20 \text{ (V)} = 40 \text{ (V)}$$

Another set of solutions: $E_1 = 20 E_2 = 20(40) = E_2 = 40 \text{ (V)}$

Ex 2.22 $\vec{r}_p(t) = (x(t), y(t))$, $\dot{x} = \frac{dx}{dt} = \frac{dx}{d\theta} \frac{d\theta}{dt}$, $\dot{y} = \frac{dy}{dt} = \frac{dy}{d\theta} \frac{d\theta}{dt}$.

Let $\dot{x} = \frac{dx}{dt}$, $\dot{y} = \frac{dy}{dt}$, $\dot{\theta} = \frac{d\theta}{dt}$, and $\vec{r} = (x, y)$.

$$\dot{\vec{r}} = \frac{d\vec{r}}{dt} = \frac{d\vec{r}}{d\theta} \frac{d\theta}{dt} \implies \begin{cases} \dot{x} = \frac{dx}{d\theta} \dot{\theta} = \dot{x}_1 \\ \dot{y} = \frac{dy}{d\theta} \dot{\theta} = \dot{y}_1 \end{cases}$$

We have

$$\dot{x}_1 = \frac{dx}{d\theta} \left[(1-\eta^2 + \eta^4) \sqrt{1-\eta^2 + \eta^4} \right]$$

$$\dot{y}_1 = \frac{dy}{d\theta} \left[-(1-\eta^2 + \eta^4) \sqrt{1-\eta^2 + \eta^4} \right],$$

$$\dot{\vec{r}}_1 = \frac{d\vec{r}}{d\theta} \dot{\theta} = \dot{\vec{r}}_1$$

Ex 2.23 $\vec{r}_1 = \vec{r}_0 \frac{1+\eta^2}{1-\eta^2}$

$\vec{r}_1 = (x_1, y_1)$, $x_1 = \frac{dx_1}{d\theta}$, $y_1 = \frac{dy_1}{d\theta}$.

$$\vec{r}_1 = \vec{r}_0 \frac{dx_1/d\theta + i dy_1/d\theta}{dx/d\theta + i dy/d\theta} \frac{d\theta/dt}{d\theta/dt}$$

$$= \vec{r}_0 \frac{\frac{dx_1/d\theta + i dy_1/d\theta}{dx/d\theta + i dy/d\theta} \frac{d\theta/dt}{d\theta/dt}}{\frac{dx/d\theta + i dy/d\theta}{dx/d\theta + i dy/d\theta} \frac{d\theta/dt}{d\theta/dt}}$$

$$= \vec{r}_0 \frac{dx_1/d\theta + i dy_1/d\theta}{dx/d\theta + i dy/d\theta}$$

Ex 2.24 (a) Given: $\vec{r}_0 = (1, 0)$ at $\theta = 0$, $\vec{r}_1 = \vec{r}_0 = (1, 0)$, $\dot{\vec{r}}_1 = \dot{\vec{r}}_0$.

$$\dot{\vec{r}}_1 = \frac{d\vec{r}_1}{dt} \dot{\theta}, \quad \dot{\vec{r}}_0 = \frac{d\vec{r}_0}{dt} \dot{\theta}$$

where $\dot{\vec{r}}_1 = \frac{d\vec{r}_1}{dt} \dot{\theta} = \frac{d\vec{r}_1}{d\theta} \frac{d\theta}{dt} = \frac{d\vec{r}_1}{d\theta} \dot{\theta}$, $\dot{\vec{r}}_0 = \frac{d\vec{r}_0}{dt} \dot{\theta} = \frac{d\vec{r}_0}{d\theta} \dot{\theta}$.

$$\therefore \dot{\vec{r}}_1 = \frac{d\vec{r}_1}{d\theta} \dot{\theta} = \frac{d\vec{r}_0}{d\theta} \dot{\theta} \implies \frac{d\vec{r}_1}{d\theta} = \frac{d\vec{r}_0}{d\theta}$$

$$\dot{\vec{r}}_1 = \frac{d\vec{r}_1}{d\theta} \dot{\theta} = \frac{d\vec{r}_0}{d\theta} \dot{\theta} \implies \frac{d\vec{r}_1}{d\theta} = \frac{d\vec{r}_0}{d\theta}$$

Setting $\vec{r}_1 = \vec{r}_0$ and $\dot{\vec{r}}_1 = \dot{\vec{r}}_0$ in the first and second

we have $\dot{\vec{r}}_1 = \dot{\vec{r}}_0 = \frac{d\vec{r}_1}{d\theta} \dot{\theta} = \frac{d\vec{r}_0}{d\theta} \dot{\theta}$ ($\dot{\theta} = \frac{d\theta}{dt} = 1$)

$$= \frac{d\vec{r}_0}{d\theta} \dot{\theta}$$

$$\dot{\vec{r}}_1 = \dot{\vec{r}}_0 = \frac{d\vec{r}_0}{d\theta} \dot{\theta} = \frac{d\vec{r}_0}{d\theta} \dot{\theta}$$

$$44. \beta = \frac{1-0.9}{1+0.9} = 0.$$

$$\begin{aligned} 45. \langle E_{\text{tot}} \rangle &= \frac{1}{2} E_0 \langle \psi | E^2 | \psi \rangle = \frac{1}{2} \left(\frac{1}{\sqrt{2}} \right) \left(\frac{1}{\sqrt{2}} \right) \langle \psi | E^2 | \psi \rangle = \frac{1}{2} \langle E^2 \rangle \langle \psi | \psi \rangle \\ &= \frac{1}{2} \langle E^2 \rangle \langle \psi | \psi \rangle, \quad \psi = \frac{1}{\sqrt{2}} e^{ikx}, \quad E = \frac{1}{\sqrt{2}} e^{ikx} \\ &\implies \langle E_{\text{tot}} \rangle = \frac{1}{2} \langle E^2 \rangle = \frac{1}{2} \langle E^2 \rangle \langle \psi | \psi \rangle. \end{aligned}$$

$$\begin{aligned} \text{Prob. 21.} \quad \psi(x) &= \psi_1 e^{ik_1 x} + \psi_2 e^{ik_2 x}, \quad E(x) = \frac{1}{2} (\psi_1^2 e^{2ik_1 x} + \psi_2^2 e^{2ik_2 x}) \\ \langle E_{\text{tot}} \rangle &= \langle E^2 \rangle = \frac{1}{2}, \quad \langle E_{\text{tot}} \rangle = \langle E^2 \rangle = \frac{1}{2} \end{aligned}$$

$$46. \langle E_{\text{tot}} \rangle = \frac{1}{2} E_0 \langle \psi | E^2 | \psi \rangle = \frac{1}{2} \langle E^2 \rangle \langle \psi | \psi \rangle = \frac{1}{2} \langle E^2 \rangle.$$

$$\begin{aligned} 47. \langle E \rangle &= \frac{1}{2} E_0 \langle \psi | E | \psi \rangle = \frac{1}{2} E_0 \langle \psi | E | \psi \rangle = \frac{1}{2} E_0 \langle \psi | E | \psi \rangle \\ &= \frac{1}{2} E_0 \langle \psi | E | \psi \rangle = \frac{1}{2} E_0 \langle \psi | E | \psi \rangle = \frac{1}{2} E_0 \langle \psi | E | \psi \rangle \\ &= \frac{1}{2} E_0 \langle \psi | E | \psi \rangle = \frac{1}{2} E_0 \langle \psi | E | \psi \rangle = \frac{1}{2} E_0 \langle \psi | E | \psi \rangle \end{aligned}$$

$$48. \langle E \rangle = \frac{1}{2} E_0 \langle \psi | E | \psi \rangle = \frac{1}{2} E_0 \langle \psi | E | \psi \rangle = \frac{1}{2} E_0 \langle \psi | E | \psi \rangle$$

$$\begin{aligned} 49. \langle E_{\text{tot}} \rangle &= \frac{1}{2} E_0 \langle \psi | E^2 | \psi \rangle = \frac{1}{2} E_0 \langle \psi | E^2 | \psi \rangle = \frac{1}{2} E_0 \langle \psi | E^2 | \psi \rangle \\ E &= \frac{1}{2} E_0 \langle \psi | E | \psi \rangle = \frac{1}{2} E_0 \langle \psi | E | \psi \rangle = \frac{1}{2} E_0 \langle \psi | E | \psi \rangle \\ E &= \frac{1}{2} E_0 \langle \psi | E | \psi \rangle = \frac{1}{2} E_0 \langle \psi | E | \psi \rangle = \frac{1}{2} E_0 \langle \psi | E | \psi \rangle \end{aligned}$$

$$\begin{aligned} \text{Prob. 22.} \quad \psi_1 &= 1, \quad \psi_2 = \frac{1}{\sqrt{2}} e^{ikx} = \frac{1}{\sqrt{2}} e^{ikx} = \frac{1}{\sqrt{2}} e^{ikx} \\ E &= \frac{1}{2} E_0 \langle \psi | E | \psi \rangle = \frac{1}{2} E_0 \langle \psi | E | \psi \rangle = \frac{1}{2} E_0 \langle \psi | E | \psi \rangle \end{aligned}$$

$$\begin{aligned} 50. \langle E_{\text{tot}} \rangle &= \frac{1}{2} E_0 \langle \psi | E^2 | \psi \rangle = \frac{1}{2} E_0 \langle \psi | E^2 | \psi \rangle = \frac{1}{2} E_0 \langle \psi | E^2 | \psi \rangle \\ \langle E_{\text{tot}} \rangle &= \frac{1}{2} E_0 \langle \psi | E^2 | \psi \rangle = \frac{1}{2} E_0 \langle \psi | E^2 | \psi \rangle = \frac{1}{2} E_0 \langle \psi | E^2 | \psi \rangle \end{aligned}$$

$$\begin{aligned} 51. \langle E_{\text{tot}} \rangle &= \frac{1}{2} E_0 \langle \psi | E^2 | \psi \rangle = \frac{1}{2} E_0 \langle \psi | E^2 | \psi \rangle = \frac{1}{2} E_0 \langle \psi | E^2 | \psi \rangle \\ \langle E_{\text{tot}} \rangle &= \frac{1}{2} E_0 \langle \psi | E^2 | \psi \rangle = \frac{1}{2} E_0 \langle \psi | E^2 | \psi \rangle = \frac{1}{2} E_0 \langle \psi | E^2 | \psi \rangle \end{aligned}$$

$$52. \text{At the limit, } \psi = 1,$$

$$\begin{aligned} \langle E_{\text{tot}} \rangle &= \frac{1}{2} E_0 \langle \psi | E^2 | \psi \rangle \\ &= \frac{1}{2} E_0 \langle \psi | E^2 | \psi \rangle = \frac{1}{2} E_0 \langle \psi | E^2 | \psi \rangle \end{aligned}$$

$$\psi = \frac{1}{\sqrt{2}} e^{ikx}, \quad E = \frac{1}{\sqrt{2}} e^{ikx}$$

$$\langle E_{\text{tot}} \rangle = \frac{1}{2} E_0 \langle \psi | E^2 | \psi \rangle = \frac{1}{2} E_0 \langle \psi | E^2 | \psi \rangle = \frac{1}{2} E_0 \langle \psi | E^2 | \psi \rangle$$

Ex. 2.2 $F_1 = 0$, $F_2 = 1$

a) $0 < t < T/2$



b) $T/2 < t < T$



c) $T < t < 3T/2$



d) $3T/2 < t < 2T$



$$F_1^* = 0, \quad F_2^* = F_1^* = 0,$$

$$F_1^* = 0, \quad F_2^* = 0,$$

$$F_1^* = 0, \quad F_2^* = 0,$$

$$F_1^* = 0, \quad F_2^* = -F_1^* = 0,$$

$$F_1^* = 0, \quad F_2^* = 0,$$

$$F_1^* = 0, \quad F_2^* = 0,$$

At $t = 0$, both $V(t)$ and $X(t)$ revert back to the initial state at $t = 0$, and the cycle repeats itself with a period of T .

Ex. 10 $R_1 = R_2 = 1 \rightarrow C_1 = \frac{1}{2}, R_2 = 1 \rightarrow C_2 = \frac{1}{2}, T = 4\mu s$

a) Input voltages



$$U_1' = \frac{R_2}{R_1 + R_2} U_1$$

$$= \frac{1}{2} U_1$$

$$U_2' = \frac{R_1}{R_1 + R_2} U_2$$

$$U_2' = \frac{1}{2} U_2$$

b) Currents



Ex. 11 $R_1 = 2 \rightarrow C_1 = 1, R_2 = 1 \rightarrow C_2 = \frac{1}{2}, T = 4\mu s$

a) Input voltages



b) Currents



c)



Ex. 2.12 The current reflection diagram for Example 9-11 is



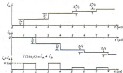
$$I_0 = \frac{1}{2} \quad I_1 = 1$$

$$T = 2 \text{ (unit)}$$

Indices on the directed lines are normalized with respect to

$$I_0^2 = \frac{I_1^2}{I_0^2} = \frac{1}{1} = 1$$

$$= 1 \text{ (unit)}$$



Ex. 2.11 Use the equivalent circuit in Fig. 2.10(b) to study transient voltage and currents:



(a) Amplitude of first current wave following from $t = 0$ to $t = 0.1$ $I_1 = \frac{V_s}{L_s + L_L} = \frac{10}{0.01 + 0.01} = 500$

$$\text{Refer to Fig. 2.10(b)} \quad L_s = 0.01, \quad L_L = 0.01$$

$$L = L_s + L_L \rightarrow I_1 = \frac{V_s}{L_s + L_L} = \frac{10}{0.02} = 500, \quad I_2 = 0 \quad T = 0.02$$



$$L = 0.02 \rightarrow I_1 = \frac{10}{0.02} = 500, \quad I_2 = 0 \quad T = 0.02$$

Ex. 2.12 (a) Governing equation at the load for $t > T$

$$L_L \frac{di_L(t)}{dt} + (R_s + R_L)i_L(t) = 0 \quad t > T$$

$$\text{Solution: } i_L(t) = \frac{V_s}{L_s + L_L} \left[1 - e^{-\frac{R_s + R_L}{L_s + L_L}(t - T)} \right], \quad t > T$$

For the present problem, $V_s = 10$ V, $R_s = 0.01 \Omega$, $R_L = 0.01 \Omega$

$$L_s = 0.01 \text{ H}, \quad L_L = 0.01 \text{ H}, \quad T = 0.01 \text{ s} = 10 \text{ ms}$$

$$i_L(t) = \frac{10}{0.02} \left[1 - e^{-\frac{0.02}{0.02}(t - 0.01)} \right], \quad t > 0.01 \text{ s}$$

$$v_L(t) = 0.01 \frac{di_L(t)}{dt} = \frac{10}{2} \left[1 - e^{-\frac{0.02}{0.02}(t - 0.01)} \right], \quad t > 0.01 \text{ s}$$





At $t = 40 \text{ ms}$,

$$\dot{x}(40) = \frac{1}{2} [1 - e^{-2(40-20)}] =$$

$$= 0.19 \text{ m/s}$$

Ex-20 From Eq (9-27c) $x(20) = 20^2 - \frac{1}{2} (20)^2$ ①

At the end: $\dot{x}(20) = \frac{dx}{dt} = 20$ ②

Substituting ② in ① $20 \frac{dx}{dt} = (\frac{1}{2} - \frac{1}{2}) (20)^2 = \frac{1}{2} (20)^2$ ③

a) Solution of ③: $x(20) = 20^2 \frac{dx}{dt} [1 - e^{-2(20-20)}] = 0$

For this problem: $x^* = \frac{20}{2} = 20 \text{ cm}$, $\dot{x}_0 = \frac{dx}{dt} = 20$

$T = 20 \text{ ms}$, $\dot{x}_0 = 20 \text{ m/s}$, $\dot{x}_0 = 2.0 \times 10^{-2} \text{ m/s}$

$x(20) = 20 \times [1 - e^{-2(20-20)}] \text{ cm}$, $x = 20 \text{ cm}$

From ③: $\dot{x}(20) = 20 \text{ m/s} = 2.0 \times 10^{-2} \text{ m/s}$

b) At $t = 40 \text{ ms}$, $x(40) = 20 \text{ cm}$, $\dot{x}(40) = 0.19 \text{ m/s}$



Ex-21 a) $\dot{x}_1 = \dot{x}_2 = \dot{x}_3 = \left(\frac{1}{2} \right) e^{t/2} \text{ cm/s}$ Since $\dot{x}_1 = \dot{x}_2 = \dot{x}_3$ then

$\dot{x}_1 = \sqrt{\frac{2 \times 10^{-2}}{2 \times 10^{-2}}} = \left(\frac{1}{2} \right) e^{t/2} \text{ cm/s}$ Since $\dot{x}_1 = \dot{x}_2 = \dot{x}_3$ then $\dot{x}_1 = \dot{x}_2 = \dot{x}_3$

$\therefore \dot{x}_1 = \frac{1}{2} = \left(\frac{1}{2} \right) e^{t/2} = \left(\frac{1}{2} \right) e^{t/2}$ and $t = 0$ then $\dot{x}_1 = \dot{x}_2 = \dot{x}_3$

b) $\left| \frac{\dot{x}_1}{\dot{x}_2} \right| = \left| \frac{1}{2} \right| = \left(\frac{1}{2} \right) e^{t/2} = \left(\frac{1}{2} \right) e^{t/2}$ and $t = 0$ then $\dot{x}_1 = \dot{x}_2 = \dot{x}_3$

c) $\frac{1}{2} = \frac{1}{2} = 1$ then $\left| \frac{\dot{x}_1}{\dot{x}_2} \right| = 1$ and $\dot{x}_1 = \dot{x}_2 = \dot{x}_3$

Then $\left| \frac{\dot{x}_1}{\dot{x}_2} \right| = 1$ and $\dot{x}_1 = \dot{x}_2 = \dot{x}_3$

P.20-21 $\vec{r} = 2 \cos^2(16t) \hat{i} + 2 \sin^2(16t) \hat{j}$

- a) Given directed line, $\vec{r} = 2 \cos^2(16t) \hat{i} + 2 \sin^2(16t) \hat{j}$

Sketch curve: Start from $\vec{r}_0$ on the extreme right, rotate clockwise one complete revolution (2π or 360°) and continue on $\vec{r}$ as additional 2π or 4π to return to the "unwinding" toward generator "axis". Round $\vec{r} = 2 \cos^2(16t) \rightarrow \vec{r}_1 = 2\pi = 2(2\pi \cos^2(16t)) = 2\pi \cos^2(16t)$

Draw a straight line from the (0, 2π) point through the center and intercept at (2, 2π) on the opposite side of the circle $\rightarrow \vec{r}_2 = \frac{2\pi}{2} = (\pi \cos^2(16t)) = \pi \cos^2(16t)$ (2).

- b) Given directed line, $\vec{r} = 2 \cos^2(16t) \hat{i} + 2 \sin^2(16t) \hat{j}$

Start from the extreme-left point $\vec{r}_0$, rotate clockwise one complete revolution and continue on $\vec{r}$ as additional 2π or 4π to round $\vec{r} = 2\pi \cos^2(16t) \rightarrow \vec{r}_1 = 2\pi = 2(2\pi \cos^2(16t)) = 2\pi \cos^2(16t)$ (2).

Draw a straight line from the (0, 2π) point through the center and intercept at (2, 2π) on the opposite side of the circle $\rightarrow \vec{r}_2 = \frac{2\pi}{2} = (\pi \cos^2(16t)) = \pi \cos^2(16t)$ (2).

Circle



$$\vec{r}_2 = \frac{2\pi}{2} (\cos^2(16t)) = \pi \cos^2(16t)$$

- c) 1. Locate $\vec{r}_2 = \pi \cos^2(16t)$ on Sketch curve (Sketch $\vec{r}_1$).

2. Find center of $\vec{r}$ above a 2π-circle through $\vec{r}_1$, intercepting $\vec{r}_2$ at 1π. $\rightarrow \vec{r} = 1 \cos^2(16t)$

d) $\vec{r} = \frac{2\pi \cos^2(16t)}{2\pi \cos^2(16t)} \cos^2(16t) = \cos^2(16t)$

- e) 1. Draw line $\vec{r}_2$, intercepting the periphery of $\vec{r}_1$.

Round 2π or 4π as "unwinding" toward generator "axis".

2. Move clockwise by 2π or 4π to start (Sketch $\vec{r}_1$).

3. Note $\vec{r}$ and $\vec{r}_1$, intercepting the (0, 2π) at $\vec{r}_2$.

4. Round $\vec{r}_2 = \pi \cos^2(16t)$ at $\vec{r}_1$.

$$\vec{r}_1 = 2\pi \vec{r}_2 = 2\pi (\pi \cos^2(16t))$$

4) Extend line $Q_1'Q_2$ to Q_3 . Read $Q_3 = 0.007 \text{ cm}$.

$$Q_1 = \frac{1}{2} Q_3 = 0.0035 \text{ cm} \quad (2)$$

Q There is no voltage minimum on this line, but $Q_1 < Q_2$.

Ex. 2.11



$$Q_1 = \frac{1}{2} (1 + |\Gamma|) = 0.00125$$

4) Locate $Q_1 = 0.00125$ on Smith chart (Point Q_1). Move counter-clockwise along a $|V|$ -circle through Q_1 , intersecting line 0.25λ at 1.75λ . — $z = 0.75\lambda$.

$$b) \quad I = 0.418 \text{ A} \quad (1)$$

4) i. Draw line 0.75λ , intersecting the periphery at Q_2 . Read 0.404 as "normalized forward power" scale.

ii. Move clockwise by 0.125λ to 0.875 (Point Q_2).

3. Plot 0 and Q_2 , intersecting the $|V|$ -circle at Q_1 .

ii. Read $Q_1 = 0.017$ at Q_1 .

$$Q_1 = 0.017 Q_2 = 0.007 \times 0.125 = 0.0021$$

4) i. Extend line $Q_1'Q_2$ to Q_3 . Read $Q_3 = 0.017$ at Q_3 .

$$Q_1 = \frac{1}{2} Q_3 = 0.0085 \text{ cm} \quad (2)$$

Q There is a voltage minimum at $Q_1 = 0.0085 \text{ cm}$.

Ex. 2.12 $\lambda/8 = 20$, $\lambda = 160 \text{ mm}$

Find voltage minimum occurs at $Q_1 = \frac{1}{2} Q_2 = 0.1 \text{ cm}$.



Q i. Short from Q_2 and rotate counter-clockwise 0.125λ toward the load to Q_1 .

ii. Draw the $|V|$ -circle, intersecting line 0.25λ at Q_2 (Point).

3. Plot 0 and Q_2 , intersecting the $|V|$ -circle at Q_1 .

4. Find $\vec{r}_2 = x_2\hat{i} + y_2\hat{j}$.

$\vec{r}_2 = 3.0\hat{i} - 4.0\hat{j}$ cm.

5) $\vec{r} = \frac{1}{\sqrt{2}}(\hat{i} + \hat{j})$ and $\vec{r}' = \frac{1}{\sqrt{2}}(\hat{i} - \hat{j})$.

- 6) If $\vec{r}_2 = 0$, the first charge minimum could be at $\vec{r}_1 = x_1\hat{i} = 1.5$ cm from the short-charge.

2.10.10



a) $\vec{r}_1 = \frac{1}{\sqrt{2}}(1.0 - j1.0)$
 $= 0.71\hat{i} - 0.71\hat{j}$ m.

7. Define $\vec{r}_1$ as field point (Point P).

8. Define $\vec{r}$ and $\vec{r}'$ and extend to $\vec{r}$.

9. Find an "arbitrary" fixed parameter "value" $d = 1.0$ m.

$\vec{r}_1 = 0.71\hat{i} - 0.71\hat{j}$ m, $d = 1.0$ m, $\vec{r} = \frac{1}{\sqrt{2}}(\hat{i} + \hat{j})$ m.

$\frac{d\vec{r}}{dt} = 0.71\hat{i} - 0.71\hat{j}$ m/s, $d = \frac{1}{\sqrt{2}} \ln \frac{1}{\sqrt{2}} = 0.71$ (Eq. 10).



10) 1. Define $\vec{r}_1$ as field point (Point P).

2. Draw line from $\vec{r}$ through $\vec{r}_1$ to $\vec{r}'$. Find an "arbitrary" fixed parameter "value" $d = 1.0$ m.

3. Define extension $d_2 = 1.0$ m to 1.0 m (Point P).

4. Find $\vec{r}_1$ intersecting the $\vec{r}$ line through $\vec{r}$ at P.

5. Find point P on line $\vec{r}$ such that $\frac{d\vec{r}}{dt} = 0.71$.

6. Find at P: $\vec{r}_1 = 0.71\hat{i} - 0.71\hat{j}$ m, $\vec{r}_2 = 0.71\hat{i} - 0.71\hat{j}$ m.

- 4) d. Obtain relationships from $\mathcal{L}_1$ in the following format:
 "generator" equals to $a \cdot 10^b$, say P^c .

1. From 10^0 .

2. Obtain pairs β and β' such that

$$\beta\beta' = 10^{mP} 10^0 = 0.001 \cdot 10^0$$

3. Round off P : $\beta_1 = 0.001 + \beta_2 \cdot 10^0 \rightarrow \beta_1 = 0.1 + \beta_2 \cdot 10^{-1}$.

$$\text{E.g.: } \beta = 2 \times 10^2 \text{ (bad)}, \quad \beta = 2.2 \text{ (bad)} \rightarrow \beta = \frac{22}{10} = 2.2 \text{ (good)}$$

$$\beta_2 = 73 + 100 = 173 \text{ (bad)}$$

For four-wise truncation lines: $\beta_1 = \text{the last } 4 \left(\frac{1}{10} \right)$

$$\beta = 2 \text{ (bad)} \rightarrow \beta_1 = 0.002 \text{ (bad)}$$

E.g.: β_1



$$\beta_1 = 0.002 \text{ (bad)}$$

$$\beta_2 = 1 \times 10^0$$

4) For truncation:

$$\beta_1 = 0.002 \times 10^0$$

$$\beta_2 = 0.1 + 0.01 = 0.11 \times 10^0$$

$$\beta_3 = 0.1 \times 10^0$$

$$\beta_4 = 0.001 \times 10^0$$

$$\beta_5 = 0.001 \times 10^0$$

$$\beta_6 = 0.001 \times 10^0$$

4) For $\beta_1 = 0.002 \times 10^0$, $\beta_2 = 0.11 \times 10^0$.

The required truncated sub-numbers are $\beta_1 = 0.002 \times 10^0$.

	$\beta_1 = 0.002 \times 10^0$	$\beta_2 = 0.11 \times 10^0$
$\beta_1 = 0.002 \times 10^0$	$\beta_1 = 0.002 \times 10^0$	$\beta_2 = 0.11 \times 10^0$
$\beta_2 = 0.11 \times 10^0$	$\beta_1 = 0.002 \times 10^0$	$\beta_2 = 0.11 \times 10^0$

Ex. 3.13 $\beta = 1$ and $\alpha = j\omega$

Use Smith chart as an impedance chart. Same restriction as that in problem 3-12 except R_{in} would be in the leftmost half (normalized by Z_0) and jB_{in} would be in the bottom.

$$\underline{Z}_1: Z_1 = j\omega L = j\omega Z_0 \text{ with } Z_0 = 50 \Omega \text{ and } \omega = 100 \text{ Mrad/s.}$$

$$\underline{Z}_2: Z_2 = 1 - j1 \text{ with } Z_0 = 50 \Omega \text{ and } \omega = 100 \text{ Mrad/s.}$$

To obtain a match with a series stub having $Z_0 = \frac{50}{2} \Omega$, we need a normalized stub impedance of $j\frac{50}{2} = +j25$ for solution corresponding to $\underline{Z}_1$. From Smith chart we obtain the required stub length $L_1 = 0.388 \lambda$.

Similarly, for solution corresponding to $\underline{Z}_2$, a stub with a normalized impedance of $j10$ is needed, which requires a stub length $L_2 = 0.162 \lambda$.

Ex. 3.14



$$Z_1 = 0.5 + j1 \Omega$$

$$\underline{Z}_1: Z_1 = 0.5 + j1 \Omega \text{ (normalized to } Z_0)$$

$$\underline{Z}_2: Z_2 = 1 - j1 \Omega \text{ (normalized to } Z_0)$$

$$\underline{Z}_3: Z_3 = 0.5 \Omega \text{ (normalized to } Z_0)$$

$$\underline{Z}_4: Z_4 = 1 \Omega \text{ (normalized to } Z_0)$$

$$\underline{Z}_5: Z_5 = 0.5 \Omega \text{ (normalized to } Z_0)$$

$$\underline{Z}_6: Z_6 = 1 \Omega \text{ (normalized to } Z_0)$$

	of Normalized Stub	of Quarter-Wave Stub
$(Z_{in})_1 = Z_0 \cdot Z_1 = -j50 \Omega$	$Z_{in} = 0.5 + j1 \Omega$	$Z_{in} = 0.5 + j1 \Omega$
$(Z_{in})_2 = Z_0 \cdot Z_2 = j50 \Omega$	$Z_{in} = 1 - j1 \Omega$	$Z_{in} = 1 - j1 \Omega$
$(Z_{in})_3 = j50 \Omega$	$Z_{in} = 0.5 \Omega$	$Z_{in} = 0.5 \Omega$
$(Z_{in})_4 = -j50 \Omega$	$Z_{in} = 1 \Omega$	$Z_{in} = 1 \Omega$

Chapter 11

Waveguides and Cavity Resonators

Ex. 11-1 $\vec{E} \times \vec{E} = -j\omega\mu\vec{H}$ ①

$\vec{H} \times \vec{H} = j\omega\epsilon\vec{E}$ ②

From ①, $(\vec{E}_1 + \vec{E}_2) \times (\vec{E}_1 + \vec{E}_2) = -j\omega\mu(\vec{H}_1 + \vec{H}_2)$

—— $\vec{E}_1 \times \vec{E}_1 + \vec{E}_2 \times \vec{E}_2 = -j\omega\mu(\vec{H}_1 + \vec{H}_2)$

—— $\vec{E}_1 \times \vec{E}_2 + \vec{E}_2 \times \vec{E}_1 = -j\omega\mu\vec{H}_1$ ③
 $(\because \vec{E}_1 \times \vec{E}_2 + \vec{E}_2 \times \vec{E}_1 = 2\vec{E}_1 \times \vec{E}_2)$

Similarly from ② we obtain

$\vec{E}_1 \times \vec{E}_2 + \vec{E}_2 \times \vec{E}_1 = j\omega\epsilon(\vec{H}_1 + \vec{H}_2)$ ④

Combining ③ and ④, we have

$j\omega\epsilon(\vec{H}_1 + \vec{H}_2) = 2\vec{E}_1 \times \vec{E}_2 + j\omega\mu(\vec{H}_1 + \vec{H}_2)$

—— $\vec{H}_1 = -\frac{1}{\mu} (2\vec{E}_1 \times \vec{E}_2 - \epsilon\mu j\omega\vec{H}_1)$, etc. i.e. eqn.

Similarly, $\vec{H}_2 = -\frac{1}{\mu} (2\vec{E}_1 \times \vec{E}_2 - \epsilon\mu j\omega\vec{H}_2)$

Ex. 11-2 (a) $\omega/\omega_c = \omega/\omega_c$



From Eq. (11-28):

$$\left(\frac{k}{k_c}\right)^2 + \left(\frac{\omega}{\omega_c}\right)^2 = 1$$

From Eq. (11-27):

$$\left(\frac{\omega}{\omega_c}\right)^2 + \left(\frac{k}{k_c}\right)^2 = 1$$

Both are equivalent of a unit circle.

(b)

ω/ω_c or ω/ω_c



From Eq. (11-28):

$$\left(\frac{k}{k_c}\right)^2 = 1 - \frac{\omega^2}{\omega_c^2}$$

From Eq. (11-27):

$$\left(\frac{\omega}{\omega_c}\right)^2 = 1 - \frac{k^2}{k_c^2}$$



From Eq. (10-117):

$$\left(\frac{dv_o}{dt}\right) = \frac{(v_o - v_o(ss))}{(T_0/2)} = -1$$

$$\Rightarrow \text{At } t/T_0 = 1.07:$$

$$v_o(t) = 1.4 \text{ V}$$

$$v_o(ss) = 0.6 \text{ V}$$

$$\beta_1 T_0 = 0.4 \text{ s}$$

$$A_{v1}(s) = 1.4 \text{ V}$$

Ex. 10-11 a) For parallel-plate capacitors:



$$v_o(t) - v_o = \left(\frac{v_o(ss)}{T_0}\right)^2$$

$$v_o = \frac{v_o(ss)}{2.4}$$

$$v_o = \frac{v_o(ss)}{2.4}$$

$$v_o = \frac{v_o(ss)}{2.4}$$

At low-frequency parameter β and μ affect both v_o and the slope of the $v_o(t)$ curves; it affects v_o

but not the slope at high-frequency. b) Yes.

Ex. 10-12 Find expressions for T_0 and β , from Eq. (10-117):

$$C_1^* v_o = A_1 \sin(\omega_p t)$$

$$C_1^* v_o = \frac{A_1}{\omega_p} \cos(\omega_p t)$$

$$C_1^* v_o = -\frac{A_1}{\omega_p} \sin(\omega_p t)$$

Surface charge densities:

$$Q_1 = Q_0 + \epsilon \left| \frac{dv_o}{dt} \right| = \epsilon \omega_p A_1 \cos(\omega_p t)$$

$$Q_2 = Q_0 + \epsilon \left| \frac{dv_o}{dt} \right| = -\epsilon \omega_p A_1 \sin(\omega_p t)$$

Surface current densities:

$$J_1 = J_0 + \epsilon \left| \frac{dv_o}{dt} \right| = J_0 + \epsilon \omega_p A_1 \cos(\omega_p t)$$

$$J_2 = J_0 + \epsilon \left| \frac{dv_o}{dt} \right| = J_0 + \epsilon \omega_p A_1 \sin(\omega_p t)$$

P.10-1 Field expressions for H_z modes from Eqs. (10-11) and (10-12):

$$E_z^{\text{TM}} = E_0 \cos(\beta_z z) \cos(\beta_y y) \cos(\beta_x x) \cos(\omega t)$$

$$E_y^{\text{TM}} = \frac{E_0}{2} \beta_z \sin(\beta_z z) \cos(\beta_y y) \cos(\beta_x x) \cos(\omega t)$$

$$E_x^{\text{TM}} = -\frac{E_0}{2} \beta_z \sin(\beta_z z) \sin(\beta_y y) \cos(\beta_x x) \cos(\omega t)$$

$$H_z^{\text{TE}} = H_0 \cos(\beta_z z) \sin(\beta_y y) \sin(\beta_x x) \sin(\omega t)$$

$$H_y^{\text{TE}} = -H_0 \cos(\beta_z z) \cos(\beta_y y) \sin(\beta_x x) \sin(\omega t) \quad \left\{ \begin{array}{l} \beta_y \text{ for } n \text{ odd,} \\ \beta_z \text{ for } n \text{ even.} \end{array} \right.$$

P.10-2 Plot E_z and the field expressions in problem P.10-1.



P.10-3 Plot H_z and the field expressions in problem P.10-1.



P.10-4 Using the field expressions in problem P.10-1, show:

$$\vec{E}_{\text{TM}} \cdot \vec{E}_0 = \int_V \vec{E}_{\text{TM}} \cdot \vec{E}_0 dV = \int_V (E_z E_z^* + E_y E_y^* + E_x E_x^*) dV$$

$$\vec{E}_{\text{TM}} \cdot \vec{E}_0 = \int_V \vec{E}_{\text{TM}} \cdot \vec{E}_0 dV = \frac{E_0^2}{2} \int_V \cos^2(\beta_z z) \cos^2(\beta_y y) \cos^2(\beta_x x) dV$$

$$(\vec{E}_{\text{TM}} \cdot \vec{E}_0)_{\text{avg}} = \int_V \vec{E}_{\text{TM}} \cdot \vec{E}_0 dV = \frac{E_0^2}{2} \int_V \cos^2(\beta_z z) \cos^2(\beta_y y) \cos^2(\beta_x x) dV$$

$$(\vec{E}_{\text{TM}} \cdot \vec{E}_0)_{\text{avg}} = \frac{E_0^2}{2} \int_V \cos^2(\beta_z z) \cos^2(\beta_y y) \cos^2(\beta_x x) dV$$

$$(\vec{E}_{\text{TM}} \cdot \vec{E}_0)_{\text{avg}} = \int_V (\vec{E}_{\text{TM}} \cdot \vec{E}_0)_{\text{avg}} dV = \frac{E_0^2}{2} \int_V \cos^2(\beta_z z) \cos^2(\beta_y y) \cos^2(\beta_x x) dV$$

$$\text{From Eq. (10-10)} \quad \vec{E}_{\text{TM}} = \frac{E_0}{2} \left(\cos(\beta_z z) \cos(\beta_y y) \cos(\beta_x x) + \cos(\beta_z z) \cos(\beta_y y) \cos(\beta_x x) \right)$$

which is the same as Eq. (10-11)

Prob 10 Given: $\vec{r} = 2.00\hat{i} + 1.00\hat{j}$ (m), $\vec{v} = 3.00\hat{i}$, $\mu_0 = 1$,
 $\vec{r} = 10^{-10}$ (cm), $\vec{A} = 2.00\hat{j}$ (m), $\vec{p} = 10^{-20}$ (cm).

a) Field inside

$$\vec{B} = \mu_0 \vec{J} \times \vec{r} = 3.00\hat{j} \times 1.00\hat{j} = 0$$

$$B_x = \frac{\mu_0}{4\pi} \frac{\partial B_z}{\partial y} = 1.00\hat{j} \times 10^{-20} \text{ (cm)}.$$

$$B_y = \frac{\mu_0}{4\pi} \frac{\partial B_z}{\partial x} = 2.00\hat{j} \times 10^{-20} \text{ (cm)}.$$

$$B_z = \mu_0 \vec{J} \times \vec{r} = \frac{\partial B_z}{\partial x} = 0 \times 10^{-20} \text{ (cm)}.$$

$$B_x = 0 = \frac{\partial B_z}{\partial y} = 0 \times 10^{-20} \text{ (cm)}.$$

b) Field inside — $(\vec{B})_{\text{inside}} = \frac{\mu_0 \vec{J}}{2\pi R} = 2 \times 10^{-20} \text{ (cm)} = \vec{B}$.

$$\vec{B} = \mu_0 \sqrt{1 - \frac{v^2}{c^2}} \vec{J} = 0.999\hat{j}.$$

$$\vec{B} = \mu_0 \sqrt{1 - \frac{v^2}{c^2}} \vec{J} = 0.999\hat{j} \times 10^{-20} \text{ (cm)}.$$

$$B_x = \frac{\mu_0}{4\pi} \frac{\partial B_z}{\partial y} = 1.00\hat{j} \times 10^{-20} \text{ (cm)}.$$

$$B_y = \frac{\mu_0}{4\pi} \frac{\partial B_z}{\partial x} = \frac{\mu_0}{2\pi} \frac{\partial B_z}{\partial x} = 2.00\hat{j} \times 10^{-20} \text{ (cm)}.$$

$$B_z = \mu_0 \vec{J} \times \vec{r} = 0.000 \times 10^{-20} \text{ (cm)}.$$

$$B_x = 0 = \frac{\partial B_z}{\partial y} = 0.000 \times 10^{-20} \text{ (cm)}.$$

$$B_y = \mu_0 \vec{J} \times \vec{r} = 2.000 \times 10^{-20} \text{ (cm)}.$$

c) Field inside — $(\vec{B})_{\text{inside}} = \frac{\mu_0 \vec{J}}{2\pi R} = 2 \times 10^{-20} \text{ (cm)} = \vec{B}$.

$$\vec{B} = \mu_0 \sqrt{1 - \frac{v^2}{c^2}} \vec{J} = 0.999\hat{j}.$$

$$\vec{B} = \mu_0 \sqrt{1 - \frac{v^2}{c^2}} \vec{J} = 0.999\hat{j} \times 10^{-20} \text{ (cm)}.$$

$$B_x = \frac{\mu_0}{4\pi} \frac{\partial B_z}{\partial y} = 1.00\hat{j} \times 10^{-20} \text{ (cm)}.$$

$$B_y = \frac{\mu_0}{4\pi} \frac{\partial B_z}{\partial x} = \frac{\mu_0}{2\pi} \frac{\partial B_z}{\partial x} = 2.00\hat{j} \times 10^{-20} \text{ (cm)}.$$

$$B_z = \mu_0 \vec{J} \times \vec{r} = 0.000 \times 10^{-20} \text{ (cm)}.$$

$$B_x = 0 = \frac{\partial B_z}{\partial y} = 0.000 \times 10^{-20} \text{ (cm)}.$$

$$B_y = \mu_0 \vec{J} \times \vec{r} = 2.000 \times 10^{-20} \text{ (cm)}.$$

Prob 11 a) Field inside — $(\vec{B})_{\text{inside}} = (\vec{B})_{\text{outside}} = 2 \times 10^{-20} \text{ (cm)} = \vec{B}$.

All required parameters are the same as above for the Field inside in problem 10. For Field outside, except $\vec{r}$. Using $\vec{r} = 10^{-10}$ (cm), we have

$$B_x = \frac{\mu_0}{4\pi} \frac{\partial B_z}{\partial y} \left(\frac{\partial B_z}{\partial y} \right) = 1.00\hat{j} \times 10^{-20} \text{ (cm)}.$$

b) TE₁₀ mode ——— $(\beta_z)_{TE_1} = (\beta_z)_{TE_0} = 4\pi \times 10^6 \text{ (rad/m)}$ < β .

All required quantities are the same as those for the TE₀ mode in problem 8.10.1 (a), except α_1 .

$$\alpha_1 = \frac{1}{2L} \int_0^{2L} \sqrt{\frac{\epsilon_1}{\epsilon_0}} \left(\frac{\beta}{\beta_0} \right)^2 dz = 2.44 \times 10^{-3} \text{ (np/m)}.$$

8.10.2 For TE₁₀ mode in a parallel-plate waveguide,

$$\begin{aligned} \alpha_1 &= \frac{1}{2L} \int_0^{2L} \sqrt{\frac{\epsilon_1}{\epsilon_0}} \frac{1}{\sqrt{1 - \beta_z^2/\beta_0^2}} dz \\ &= \frac{1}{2L} \int_0^{2L} \sqrt{\frac{\epsilon_1}{\epsilon_0}} \frac{1}{\beta \cos \theta} dz, \end{aligned}$$

$$\text{where } \beta \cos \theta = \alpha = \alpha^2, \quad \alpha = L/L_0.$$

a) To find minimum α_1 , set

$$\frac{d\alpha_1}{d\alpha} = 1 + 2\alpha^2 \alpha^{-3} = 0 \quad \rightarrow \quad \alpha = \frac{1}{\sqrt{2}}.$$

$$\therefore L = \frac{1}{\sqrt{2}} L_0.$$

$$\text{b) At } L/L_0 = 1/\sqrt{2}, \quad \frac{1}{\beta \cos \theta} = 1.414,$$

$$\text{and} \quad \min \alpha_1 = \frac{1}{2L} \int_0^{2L} \sqrt{\frac{\epsilon_1}{\epsilon_0}} \frac{1}{\beta \cos \theta} dz.$$

c) For $\epsilon_1 = 2.44 \times 10^3 \text{ (rad/m)}$, $\beta = 4\pi \times 10^6 \text{ (rad/m)}$, $\beta_0 = 1.257 \times 10^8 \text{ (rad/m)}$,
and $\mu_0 = 4\pi \times 10^{-7} \text{ (H/m)}$,

$$(\beta_z)_{TE_1} = \frac{1}{\sqrt{1 - \beta_z^2/\beta_0^2}} \beta_z = 4\pi \times 10^6 \text{ (rad/m)},$$

$$\min \alpha_1 = 2.44 \times 10^{-3} \text{ (np/m)}.$$

8.10.3 Parallel-plate waveguide: identical, for ω^2/ω_0^2 .

a) TE₁₀ mode

From Eqs. 8.10.1a) and 8.10.1b)

$$\begin{cases} H_y^2 = E_z^2 \\ H_z^2 = -\frac{1}{\epsilon_0} E_y^2 \end{cases}$$

$$P_{av} = \frac{1}{2} \int_0^{2L} E_y^2 H_z^2 dz = -\frac{1}{2} \frac{1}{\epsilon_0} E_z^2.$$

$$\text{Substituting strength of wave, } \text{Atten. } E_z = 1 \text{ volt (rms)},$$

$$\text{Atten. } \left(\frac{P_{av}}{2L} \right) = \frac{1}{2\epsilon_0} (1 \text{ volt})^2 = 222 \text{ watt (rms)} = 222 \text{ (watt/m)}.$$

b) TM₀ mode

From Eq. (20-14) and (20-15)

$$\begin{cases} E_z^0(x) = E_0 \cos\left(\frac{\pi x}{a}\right) \\ E_y^0(x) = -\frac{E_0}{\sqrt{1-\beta_{TM}^2}} \sin\left(\frac{\pi x}{a}\right) \end{cases}$$

$$\beta_{TM} = \frac{1}{\sqrt{1-\beta_{TM}^2}} = 1.01 \text{ rad/m}$$

$$E_{av} = \frac{1}{a} \int_0^a E_z^0(x) dx = \frac{E_0}{a} \int_0^a \cos\left(\frac{\pi x}{a}\right) dx = \frac{E_0}{\pi \sqrt{1-\beta_{TM}^2}}$$

$$\text{Max. } \left(\frac{E_z}{E_0}\right) = \frac{1}{\pi \sqrt{1-\beta_{TM}^2}} = 1.01 \text{ rad/m (normalized)}$$

c) TE₀ mode

From Eq. (20-14) and (20-15)

$$\begin{cases} E_z^0(x) = E_0 \sin\left(\frac{\pi x}{a}\right) \\ E_y^0(x) = \frac{E_0}{\sqrt{1-\beta_{TE}^2}} \sin\left(\frac{\pi x}{a}\right) \end{cases}$$

$$E_{av} = \frac{1}{a} \int_0^a E_z^0(x) dx = \frac{E_0}{\pi \sqrt{1-\beta_{TE}^2}}$$

$$\text{Max. } \left(\frac{E_z}{E_0}\right) = \frac{1}{\pi \sqrt{1-\beta_{TE}^2}} = 1.01 \text{ rad/m (normalized)}$$



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20-13 E_z as a function of x through (20-14) through (20-17) for TM₀ mode

$$E_z^0(x, y) = \frac{2E_0}{\pi} \left(\frac{\pi}{a}\right) E_0 \cos\left(\frac{\pi x}{a}\right) \sin\left(\frac{\pi y}{b}\right)$$

$$E_z^0(x, y) = \frac{2E_0}{\pi} \left(\frac{\pi}{a}\right) E_0 \sin\left(\frac{\pi x}{a}\right) \cos\left(\frac{\pi y}{b}\right)$$

$$E_2^{\text{TE}}(x,z) = E_2 \cos\left(\frac{\pi x}{2a}\right) \sin\left(\frac{\pi z}{2b}\right),$$

$$H_2^{\text{TE}}(x,z) = \frac{2a}{\pi} \sin\left(\frac{\pi x}{2a}\right) E_2 \cos\left(\frac{\pi x}{2a}\right) \sin\left(\frac{\pi z}{2b}\right),$$

$$H_2^{\text{TM}}(x,z) = \frac{2a}{\pi} \sin\left(\frac{\pi x}{2a}\right) E_2 \cos\left(\frac{\pi x}{2a}\right) \cos\left(\frac{\pi z}{2b}\right).$$

a) Surface current densities:

$$\begin{aligned} \vec{J}_s(x=0) &= \vec{J}_s + \vec{K}|_{y=0} = \vec{J}_s + (\vec{E}_1 \hat{y} \times \vec{n}_1 + \vec{E}_2 \hat{y} \times \vec{n}_2) \\ &= \vec{J}_s \hat{y} \times \vec{n}_1 + \vec{E}_2 \frac{2a}{\pi} \sin\left(\frac{\pi x}{2a}\right) E_2 \cos\left(\frac{\pi x}{2a}\right) \hat{y} \times \vec{n}_2 \\ &= \vec{J}_s(x=0). \end{aligned}$$

$$\begin{aligned} \vec{J}_s(x=a) &= \vec{J}_s + \vec{K}|_{y=a} = \vec{J}_s + (\vec{E}_1 \hat{y} \times \vec{n}_1 + \vec{E}_2 \hat{y} \times \vec{n}_2) \\ &= \vec{J}_s \hat{y} \times \vec{n}_1 + -\vec{E}_2 \frac{2a}{\pi} \sin\left(\frac{\pi x}{2a}\right) E_2 \cos\left(\frac{\pi x}{2a}\right) \hat{y} \times \vec{n}_2 \\ &= \vec{J}_s(x=a). \end{aligned}$$



Ex 10-14 Rectangular waveguide: $a = 7$ cm, $b = 3.5$ cm.

Eq. (10-143) $\omega_{c,lm} = \frac{c}{\sqrt{\left(\frac{a}{2}\right)^2 + \left(\frac{b}{2}\right)^2}}$

Modes with the shortest $\lambda_c < 8$ cm are:

Mode	TE_{lm}	TE_{lm}	TE_{lm}	$TE_{lm}/\omega_{c,lm}$
λ (cm)	14.0	7.0	4.95	4.95

a) For $\lambda = 7$ cm, the only propagating mode is TE_{10} .

b) For $\lambda = 5$ cm, the propagating modes are:

$$TE_{10}, TE_{01}, TE_{20}, TE_{02}, \text{ and } TM_{11}.$$

Ex. 10-12 $\omega_{\text{cut}} = \omega_p \sqrt{1 - \frac{1}{\epsilon_0 \epsilon_{\infty}}}$.

For the TE_{10} mode, $k_y = \frac{\pi}{2a}$.

$\therefore \omega_{\text{cut}} = \omega_p \sqrt{1 - \frac{1}{\epsilon_0 \epsilon_{\infty}}} = \frac{\omega_p}{\sqrt{\epsilon_0 \epsilon_{\infty}}} \sqrt{1 - \frac{1}{\epsilon_0 \epsilon_{\infty}}}$.

Ex. 10-13 $G_{\perp L} = \frac{1}{\epsilon_0 \omega_p} \sqrt{\frac{\epsilon_0 \epsilon_{\infty}}{\epsilon_0 \epsilon_{\infty} - 1}} = \frac{1}{\omega_p \sqrt{\epsilon_0 \epsilon_{\infty}}} P(\omega)$.

a) $a = b$, $P(\omega) = \sqrt{\omega^2 - \omega_{\text{cut}}^2}$

b) $a = b$, $P(\omega) = \omega \sqrt{\omega^2 - \omega_{\text{cut}}^2}$.

Mode	$P(\omega)$
TE_{10}	1
$\text{TE}_{01}, \text{TE}_{10}$	1
$\text{TE}_{20}, \text{TE}_{02}$	$\sqrt{3}$
TE_{11}	4
TM_{10}	$\sqrt{3}$
TM_{01}	$\sqrt{3}$

Mode	$P(\omega)$
$\text{TE}_{20}, \text{TE}_{02}$	1
$\text{TE}_{11}, \text{TM}_{11}$	$\sqrt{3}$
$\text{TE}_{01}, \text{TE}_{10}$	1
TM_{10}	$\sqrt{3}$
TM_{01}	$\sqrt{3}$

Ex. 10-17 $f = 3 \times 10^8 \text{ Hz}$, $\lambda = 1 \text{ m}$, $a = 0.5 \text{ m}$.

Let $a = b$, $\therefore$ for TE_{10} , $G_{\perp L} = \frac{1}{\omega_p \sqrt{\epsilon_0 \epsilon_{\infty}}} \sqrt{\omega^2 - \omega_{\text{cut}}^2}$.

a) $G_{\perp L} = \frac{1}{\omega_p \sqrt{\epsilon_0 \epsilon_{\infty}}}$ for the dominant TE_{10} mode.

For $f > 0.5 G_{\perp L} = 0.25 \text{ m/s}$.

The next higher-order mode is TE_{01} with $G_{\perp L} = \frac{1}{\omega_p \sqrt{\epsilon_0 \epsilon_{\infty}}}$.

For $f < 0.5 G_{\perp L} = 0.25 \text{ m/s}$.

We choose $a = 0.5 \text{ m}$ and $b = 0.5 \text{ m}$.

b) $\omega_p = \frac{1}{\sqrt{\epsilon_0 \epsilon_{\infty}}} \sqrt{\frac{1}{\epsilon_0 \epsilon_{\infty}}} = 0.25 \times 10^8 \text{ rad/s}$.

$\lambda_p = \frac{2\pi}{\omega_p \sqrt{\epsilon_0 \epsilon_{\infty}}} = 0.157 \text{ m} = 15.7 \text{ cm}$.

$\beta = \frac{\omega}{c} = 0.5 \text{ rad/m}$.

$(G_{\perp L})_0 = \frac{1}{\omega_p \sqrt{\epsilon_0 \epsilon_{\infty}}} = 0.75 \text{ (dB)}$.

Übung 10 Gegeben: $a = 1, b = 10^2 \text{ (m)}$, $\beta = 0,3 \cdot 10^2 \text{ (m)}$, $\gamma = 0,5 \cdot 10^2 \text{ (m)}$.

a) $A = \frac{1}{2} \cdot b \cdot \sqrt{1 - \frac{\beta^2 + \gamma^2}{b^2}} = 0,433 \text{ (m)}$.

$E = \sqrt{1 - \frac{\beta^2 + \gamma^2}{b^2}} = 0,913$.

$A_1 = A \cdot E = 0,395 \text{ (m)} \cdot 0,913 = 0,361 \text{ (m)}$.

$\beta' = 2 \cdot A_1 / b_2 = 0,722 \text{ (m)}$.

$\gamma_2' = b_2 \cdot E = 0,457 \text{ (m)}$.

$\gamma_1' = a \cdot E = 0,457 \text{ (m)}$.

$(A_1)_{\text{neu}} = A_1 \cdot E = 0,331 \text{ (m)}$.

b) $A = \frac{1}{2} \cdot b \cdot \sqrt{1 - \frac{\beta^2 + \gamma^2}{b^2}} = 0,433 \text{ (m)}$.

$E = \sqrt{1 - \frac{\beta^2 + \gamma^2}{b^2}} = 0,913$.

$A_1' = A \cdot E = 0,395 \text{ (m)} \cdot 0,913 = 0,361 \text{ (m)}$.

$\beta' = 2 \cdot A_1' / b_2 = 0,722 \text{ (m)}$.

$\gamma_2' = a_2 \cdot E = 0,457 \text{ (m)}$.

$\gamma_1' = b_1 \cdot E = 0,457 \text{ (m)}$.

$(A_1)_{\text{neu}} = \frac{A_1}{E} = 0,487 \text{ (m)}$.

Übung 11 Gegeben: $a = 3 \text{ (m)}$, $b = 4 \text{ (m)}$, $\beta = 2 \text{ (m)}$.

a) $A_1 = A_2 = 0,5 \cdot a \cdot b = 6 \text{ (m)}$.

$\beta_1 = \frac{b}{a} = 1,333 \text{ (m)}$.

b) $A = \frac{1}{2} \cdot a \cdot b = 6 \text{ (m)}$, $E = \sqrt{1 - \frac{\beta^2}{a^2}} = 0,75$.

$A_2 = \frac{A}{E} = \frac{6 \text{ (m)}}{0,75} = 8 \text{ (m)}$.

c) $A_1 = \sqrt{\frac{a^2 b^2}{a^2 + b^2}} = 2,4 \text{ (m)}$, $A_2 = \sqrt{\frac{a^2 b^2}{a^2 + b^2}} = 2,4 \text{ (m)}$.

d) $a^2 = \frac{b^2}{E^2} \implies E = \frac{b}{a} = 1,333 \text{ (m)}$.

Übung 12 Gegeben: $a = 2 \text{ (m)}$, $b = 4 \text{ (m)}$, $\beta = 1 \text{ (m)}$.

a) $A = \frac{1}{2} \cdot a \cdot b = 4 \text{ (m)}$, $A_1 = A_2 = 2 \text{ (m)}$.

$\sqrt{1 - \frac{\beta^2}{a^2}} = \sqrt{1 - \frac{1^2}{2^2}} = 0,866$.

$A_2 = 2 \text{ (m)}$, $A_1 = \frac{A_2}{\sqrt{1 - \frac{\beta^2}{a^2}}} = \frac{2 \text{ (m)}}{0,866} = 2,31 \text{ (m)}$.

$a_1 = 2 \text{ (m)}$.

b) From Eqs. (20-18), (20-22) and (20-23):

$$E_z = E_0 \sin\left(\frac{\pi y}{b}\right)$$

$$H_z = -\frac{E_0}{\eta_0} \sqrt{\epsilon \left(\frac{\pi}{b}\right)} \cos\left(\frac{\pi y}{b}\right)$$

$$H_y = \sqrt{\epsilon \left(\frac{\pi}{b}\right)} \frac{E_0}{\eta_0} \sin\left(\frac{\pi y}{b}\right)$$

$$P_{av} = \frac{1}{2} \int_0^b \left(\frac{1}{\eta_0} E_0^2 \sqrt{\epsilon \left(\frac{\pi}{b}\right)} \sin^2 y \right) dy = \frac{E_0^2 b}{4 \eta_0} \sqrt{\epsilon \left(\frac{\pi}{b}\right)}$$

For $P_{av} = 10$ W at the load terminals, assuming average matched conditions:

$$|E_0|^2 = E_0 = 10,000 \text{ (vol)}, \quad |H_0| = 1000 \text{ (A)}, \quad |H_y| = 1000 \text{ (A)}$$

The waveguide is too long — The field intensities are higher at the sending end by a factor of $e^{2\alpha l} = 0.92$

$$\therefore \text{Max. } |E_0|^2 = 10,000 \text{ (vol)}$$

$$\text{Max. } |H_0| = 1113 \text{ (A)}$$

$$\text{Max. } |H_y| = 1113 \text{ (A)}$$

$$\begin{aligned} \text{d) } P_{\text{load}} = P_{\text{in}} = (E_0 H_0^* + E_0^* H_0) \frac{1}{2} \eta_0 &= -E_0 H_0^* \eta_0 = -E_0 \sqrt{\epsilon \left(\frac{\pi}{b}\right)} \frac{E_0}{\eta_0} \\ |P_{\text{load}}| &= |H_0|^2 \eta_0 = 1234 \text{ (Watt)} \end{aligned}$$

$$P_{\text{in}} = P_{\text{out}} = (E_0 H_0^* + E_0^* H_0) \frac{1}{2} \eta_0 = -E_0 H_0^* \eta_0 = -E_0 \sqrt{\epsilon \left(\frac{\pi}{b}\right)} \frac{E_0}{\eta_0}$$

$$|P_{\text{in}}| = (|E_0|^2 + |E_0^*|^2) \frac{\eta_0}{2} = \frac{E_0^2}{\eta_0} \left(\left(\frac{\pi}{b}\right)^2 + \left(\frac{\pi}{b}\right)^2 \right) \frac{\eta_0}{2} = \frac{E_0^2}{\eta_0} \left(\frac{\pi}{b}\right)^2$$

critical dimension of waveguide

$$\text{At the sending end: } \text{Max. } |P_{\text{in}}| = \frac{E_0^2}{\eta_0} \left(\frac{\pi}{b}\right)^2 = 1234 \text{ (Watt)}$$

e) Total amount of average power dissipated in 1 cm of waveguide:

$$P_d = 1000 (e^{2\alpha l} - 1) = 1000 (e^{0.08} - 1) = 24.8 \text{ (W)}$$

8.20d From problem 8.19(c), we have

$$P_{\text{av}} = \frac{E_0^2 b}{4 \eta_0} \sqrt{\epsilon \left(\frac{\pi}{b}\right)} = \frac{E_0^2}{\sqrt{\epsilon \left(\frac{\pi}{b}\right)}} = 0.767$$

$$\therefore \text{Max. } P_{\text{av}} = \frac{(10,000)^2 - (10,000 \cdot 0.92)^2}{2 \cdot 0.92} = 0.767 = 767 \text{ (W)}$$

Exercise 12.11 Let $A = \frac{1}{\sqrt{2}} \sqrt{\frac{2\pi}{\sqrt{2}}}$ and $a = \frac{\pi}{2} \in L_p$ (Exercise 12.10).

We write $\langle a, \rangle_{\text{hom}} = A \Phi(a)$, where $\Phi(a) = \frac{1 + \frac{\pi^2}{2} a^2}{2(\frac{\pi^2}{2} + 1)}$.

For $a=0$, $\langle 0, \rangle_{\text{hom}}$, we $\frac{0 + \frac{\pi^2}{2}}{2(\frac{\pi^2}{2} + 1)} = 0$.

$$\text{--- } a = \frac{\pi}{2} = \sqrt{\frac{2}{\pi}} \left[\left(1 + \frac{\pi^2}{2}\right) - \sqrt{\left(1 + \frac{\pi^2}{2}\right)^2 - \frac{\pi^2}{2}} \right]^{1/2}.$$

Exercise 12.12 Find expressions for Φ_0 made from L_p (Exercise 12.11) and (12-102) through (12-105).

$$E_0^x \langle a, \rangle = -\frac{A\pi}{2} \left(\frac{\pi}{2}\right) E_0 \sin\left(\frac{\pi}{2}\right) \cos\left(\frac{\pi}{2}\right),$$

$$E_0^y \langle a, \rangle = -\frac{A\pi}{2} \left(\frac{\pi}{2}\right) E_0 \sin\left(\frac{\pi}{2}\right) \cos\left(\frac{\pi}{2}\right),$$

$$E_0^z \langle a, \rangle = E_0 \sin\left(\frac{\pi}{2}\right) \cos\left(\frac{\pi}{2}\right),$$

$$H_0^x \langle a, \rangle = -\frac{A\pi^2}{2} \left(\frac{\pi}{2}\right) E_0 \sin\left(\frac{\pi}{2}\right) \cos\left(\frac{\pi}{2}\right),$$

$$H_0^y \langle a, \rangle = -\frac{A\pi^2}{2} \left(\frac{\pi}{2}\right) E_0 \sin\left(\frac{\pi}{2}\right) \cos\left(\frac{\pi}{2}\right).$$

Calculate a_0 from L_p (Exercise 12.11): $a_0 = \frac{\pi}{2} \sqrt{\frac{2}{\pi}}$.

$$\Phi(a) = \frac{1}{2} \int_0^a \int_0^a (E_0^x a_0' - E_0^y a_0') da_0' = \frac{a_0^2}{2} \left[\frac{2\pi}{\sqrt{2}} - \left(\frac{\pi}{2}\right) \right].$$

From problem 12.11.12:

$$\frac{\pi}{2} \langle a, \rangle = E_0 \langle a, \rangle = -E_0 \frac{A\pi}{2} \left(\frac{\pi}{2}\right) E_0 \sin\left(\frac{\pi}{2}\right) \cos\left(\frac{\pi}{2}\right),$$

$$\frac{\pi}{2} \langle a, \rangle = E_0 \langle a, \rangle = -E_0 \frac{A\pi}{2} \left(\frac{\pi}{2}\right) E_0 \sin\left(\frac{\pi}{2}\right) \cos\left(\frac{\pi}{2}\right).$$

$$E_0 \langle a, \rangle = 2 \int_0^a E_0 \langle a, \rangle_{\text{hom}} + 2 \int_0^a E_0 \langle a, \rangle_{\text{hom}}$$

$$\left[E_0 \langle a, \rangle_{\text{hom}} + \frac{1}{2} \int_0^a E_0 \langle a, \rangle_{\text{hom}} \right] da_0 = \frac{A\pi}{2} \left(\frac{\pi}{2}\right) \left(\frac{\pi}{2}\right) E_0^2$$

$$\left[E_0 \langle a, \rangle_{\text{hom}} + \frac{1}{2} \int_0^a E_0 \langle a, \rangle_{\text{hom}} \right] da_0 = \frac{A\pi}{2} \left(\frac{\pi}{2}\right) \left(\frac{\pi}{2}\right) E_0^2$$

$$E_0 \langle a, \rangle = \frac{A\pi^2 \left(\frac{\pi}{2}\right) \left(\frac{\pi}{2}\right)}{2 \left[\left(\frac{\pi}{2}\right) + \left(\frac{\pi}{2}\right) \right]} \left[\left(\frac{\pi}{2}\right) a_0 + \left(\frac{\pi}{2}\right) a_0 \right]$$

$$\therefore \langle a, \rangle_{\text{hom}} = \frac{A\pi^2 \left(\frac{\pi}{2}\right) \left(\frac{\pi}{2}\right)}{2\pi A_0 \left(\frac{\pi}{2}\right) \left(\frac{\pi}{2}\right) \left(\frac{\pi}{2}\right) \left(\frac{\pi}{2}\right)} \quad \text{At } a_0 = 0, \text{ } \langle a, \rangle_{\text{hom}} = 0$$

Ex. 22.11 a)

TM₀



b)

TE₀₁



———— Electric Field Lines

----- Magnetic Field Lines

$$c) \text{ For TM modes } E_z = \frac{b}{r} J_0\left(\frac{r}{a}\right) = \frac{b J_0\left(\frac{r}{a}\right)}{r}$$

$$\text{For TM modes, } (k)_{TM} = \frac{dE_z}{dr} \rightarrow (E_z)_{TM} = \frac{dE_z}{dr} \cdot r$$

$$\text{For TE modes, } (k)_{TE} = \frac{dH_z}{dr} \rightarrow (E_z)_{TE} = (k)_{TE} \cdot r$$

(Egneren mode)

Ex. 22.12 $\beta^2 = k^2 - k_c^2 = \omega^2 \mu \epsilon - k_c^2$

$$\text{For TE modes, } k_c = \frac{2.405}{a} \text{ for } \omega = 2\pi f \text{ and } \beta = \frac{2.405}{a} \sqrt{\mu \epsilon} \cdot \omega, \quad \omega_{c0} = 2.405/a \cdot \mu$$

$$\text{For TM modes, } k_c = \frac{2.405}{a} \text{ for } \omega = 2\pi f \text{ and } \beta = \frac{2.405}{a} \sqrt{\mu \epsilon} \cdot \omega, \quad \omega_{c0} = 2.405/a \cdot \mu$$



a) If a is doubled, k_c and ω_{c0} in diagram (a) are halved but diagram (b) will remain the same.

b) If the dielectric medium is changed then ω_{c0} and β (for $\omega > \omega_{c0}$) both ω_{c0} and β are reduced by a factor $\sqrt{\epsilon}$ and the slope of the asymptotic line is changed from k_c to $k_c/\sqrt{\epsilon}$, diagram (b) remains unchanged.

Ex. 2.11 a) For modes: $L_0^2 = L_0 L_0(\partial_0) z = 0$.

Secondary condition: $L_0^2 = 0$ at both ends and
 $g \in W$ are satisfied when n is an integer.

There are no W_{Dy} modes.

b) For modes: $L_0^2 = L_0 L_0(\partial_0) \text{ even}$, where n is an integer.
 W_{Dy} modes can exist.

c) For TE modes, i.e. it requires that $L_0(\partial_0) = 0$.
 $\rightarrow$ Eigenvalues: $(W_{\text{Dy}})_n = n_{\text{Dy}}^2/\lambda_n$, $n = 1, 2, \dots$
 For TM modes, i.e. it requires that $L_0^2(\partial_0) = 0$.
 $\rightarrow$ Eigenvalues: $(W_{\text{Dy}})_n = n_{\text{Dy}}^2/\lambda_n$, $n = 2, 3, \dots$

Ex. 2.12 From L_{Dy} (2-100) and (2-101)

Inside the slab: $\beta^2 = \omega^2 \mu_1 \epsilon_1 = k_1^2 = \omega^2 \mu_2 \epsilon_2$.

Outside the slab: $\beta^2 = \omega^2 \mu_1 \epsilon_1 = 0 = \omega^2 \mu_2 \epsilon_2$.

$$\therefore \omega_1 \sqrt{\mu_1 \epsilon_1} = \beta = \omega_2 \sqrt{\mu_2 \epsilon_2},$$

$$\text{and } \frac{\omega_1^2}{\epsilon_1 \mu_1} = \omega_2^2 = \frac{\omega_1^2}{\epsilon_2 \mu_2}.$$

Ex. 2.13 From L_{Dy} (2-100) and (2-101)

$$\left(\frac{\partial^2}{\partial x^2}\right)^2 + \left(\frac{\partial^2}{\partial y^2}\right)^2 = \left(\frac{\partial^2}{\partial x^2}\right) \left(\frac{\partial^2}{\partial y^2} + 1\right). \quad (2)$$

$$\frac{\partial^2}{\partial x^2} \left(\frac{\partial^2}{\partial y^2}\right) = \left(\frac{\partial^2}{\partial x^2}\right) \sin \left(\frac{\partial^2}{\partial y^2}\right). \quad (3)$$

Let $X = k_1 x$, $Y = \omega t$, $A = \omega_1 k_1$, and $B = \frac{\omega_1^2}{\epsilon_1 \mu_1} \frac{\omega_2^2}{\epsilon_2 \mu_2} = 1$.

$$L_{\text{Dy}} \text{ Eq. (2) becomes } \begin{cases} X^2 + Y^2 = A^2, & (4) \\ Y = AB \sin X, & (5) \end{cases}$$

a) For $\omega = 0$ and $A = \omega_1 k_1 = 0$ and

$$k_1 x = \omega t = 0 \text{ and } A = 0 \text{ and } B = 1 \text{ and } \omega_1 k_1 = 0 \text{ and } \omega_2 k_2 = 0 \text{ and } \omega_1 k_1 = 0 \text{ and } \omega_2 k_2 = 0.$$



Graphical solution:

$$X_2 = 0.0005, \quad Y_2 = 0.000001$$

$$\alpha = 0.0005 = 0.001 \text{ (approx.)}$$

$$\beta_2 = 0.0005/\alpha = 0.001 \text{ (approx.)}$$

From Eq. (5-105) $\beta = \beta_2 \sqrt{X_2 - \alpha_2^2} = 0.001 \text{ (approx.)}$

b) $f = 0.001^2 \text{ (rad)}$, $\alpha = 0.001 = 0.001 \text{ (rad)}$, $X_2/Y_2 = 0.001/0$

$$X_2 = 0.001, \quad Y_2 = 0.0000$$

$$X_2 = 0.0005, \quad Y_2 = 0.000001$$

We obtain $\alpha = 0.001 \text{ (approx.)}$

$$\beta = 0.001 \text{ (approx.)}$$

Ex. 11. From Eq. (5-105):

$$\left(\frac{dY}{dX}\right) = -\frac{X}{Y} \left(\frac{dY}{dX}\right) \cos\left(\frac{dY}{dX}\right) \quad (5)$$

Using the notation in problem 5-10-11, we obtain from equation from (5) in 5-10-11 and (5) above:

$$\begin{cases} X'' + Y'' = 0, & (6) \\ Y' = -X \sin X, & (7) \end{cases}$$

a) $f = 0.001^2 \text{ (rad)}$, $\alpha = 0.001 \text{ (rad)}$, $\alpha = 0.001 \text{ (rad)}$, $\alpha = 0.001 \text{ (rad)}$

$$X = 0.001,$$

$$Y = 0.0000,$$

$$X = 0.001,$$

$$Y = 0.0000,$$

There are no differential for curves representing Eqs. (6) and (7) above, even the curves do not exist at the given approximation.

Ex. 12. Use Eqs. (5-111) and (5-112)

$$X_2' = -\frac{dX}{dY} \frac{dY}{dX}, \quad X_2' = \frac{dX}{dY} \frac{dY}{dX}$$

$$P(X_2, Y_2) = \sin(X_2) \cos(Y_2)$$

$$R(X_2, Y_2) = \sin(X_2) \cos(Y_2)$$

Table 1:

$$\begin{aligned} \mathcal{L}_2^1(\rho) &= \mathcal{L}_2(\rho) \text{ for } \mathcal{L}_2(\rho) > 0 & \longrightarrow & \mathcal{L}_2(\rho) \text{ and } \mathcal{L}_2(\rho) \text{ are not } \rho\text{-good.} \\ \mathcal{L}_2^1(\rho) &= \frac{\mathcal{L}_2(\rho)}{\mathcal{L}_2(\rho)} \text{ for } \mathcal{L}_2(\rho) > 0 & \longrightarrow & \mathcal{L}_2(\rho) \text{ and } \frac{\mathcal{L}_2(\rho)}{\mathcal{L}_2(\rho)} \text{ are not } \rho\text{-good.} \\ \mathcal{L}_2^1(\rho) &= \frac{\mathcal{L}_2(\rho)}{\mathcal{L}_2(\rho)} \text{ for } \mathcal{L}_2(\rho) > 0 & \longrightarrow & \mathcal{L}_2(\rho) \text{ and } \frac{\mathcal{L}_2(\rho)}{\mathcal{L}_2(\rho)} \text{ are not } \rho\text{-good.} \end{aligned}$$

Table 2:

$$\begin{aligned} \mathcal{L}_2^1(\rho) &= \mathcal{L}_2(\rho) \text{ for } \mathcal{L}_2(\rho) > 0 & \longrightarrow & \mathcal{L}_2(\rho) \text{ and } \mathcal{L}_2(\rho) \text{ are not } \rho\text{-good.} \\ \mathcal{L}_2^1(\rho) &= \frac{\mathcal{L}_2(\rho)}{\mathcal{L}_2(\rho)} \text{ for } \mathcal{L}_2(\rho) > 0 & \longrightarrow & \mathcal{L}_2(\rho) \text{ and } \frac{\mathcal{L}_2(\rho)}{\mathcal{L}_2(\rho)} \text{ are not } \rho\text{-good.} \\ \mathcal{L}_2^1(\rho) &= \frac{\mathcal{L}_2(\rho)}{\mathcal{L}_2(\rho)} \text{ for } \mathcal{L}_2(\rho) > 0 & \longrightarrow & \mathcal{L}_2(\rho) \text{ and } \frac{\mathcal{L}_2(\rho)}{\mathcal{L}_2(\rho)} \text{ are not } \rho\text{-good.} \end{aligned}$$

Table 3:

$$\begin{aligned} \mathcal{L}_2^1(\rho) &= \mathcal{L}_2(\rho) \text{ for } \mathcal{L}_2(\rho) > 0 & \longrightarrow & \mathcal{L}_2(\rho) \text{ and } \mathcal{L}_2(\rho) \text{ are not } \rho\text{-good.} \\ \mathcal{L}_2^1(\rho) &= \frac{\mathcal{L}_2(\rho)}{\mathcal{L}_2(\rho)} \text{ for } \mathcal{L}_2(\rho) > 0 & \longrightarrow & \mathcal{L}_2(\rho) \text{ and } \frac{\mathcal{L}_2(\rho)}{\mathcal{L}_2(\rho)} \text{ are not } \rho\text{-good.} \\ \mathcal{L}_2^1(\rho) &= \frac{\mathcal{L}_2(\rho)}{\mathcal{L}_2(\rho)} \text{ for } \mathcal{L}_2(\rho) > 0 & \longrightarrow & \mathcal{L}_2(\rho) \text{ and } \frac{\mathcal{L}_2(\rho)}{\mathcal{L}_2(\rho)} \text{ are not } \rho\text{-good.} \end{aligned}$$

Remark 4) From Table 1 and Table 2, it is seen that $\mathcal{L}_2(\rho)$ for $\mathcal{L}_2(\rho) > 0$ is not ρ -good, which is the dominant mode.

From Eq. (10-101):

$$\alpha = \frac{\mathcal{L}_2(\rho)}{\mathcal{L}_2(\rho)} \text{ for } \mathcal{L}_2(\rho) > 0 \text{ and } \mathcal{L}_2(\rho) > 0.$$

Multiplying the α term in Eq. (10-101):

$$\beta^2 = \alpha^2 \mathcal{L}_2(\rho) = \alpha^2 \mathcal{L}_2(\rho) \text{ and } \beta = \alpha \mathcal{L}_2(\rho) = \mathcal{L}_2(\rho).$$

From Eq. (10-102): $\mathcal{L}_2^1(\rho) = \mathcal{L}_2(\rho) = \beta^2 = \mathcal{L}_2(\rho)$

$$\therefore \alpha = \frac{\mathcal{L}_2(\rho)}{\mathcal{L}_2(\rho)} = 1.$$

2) Also from Eq. (10-101), $\mathcal{L}_2(\rho) = \mathcal{L}_2(\rho)$, $\mathcal{L}_2(\rho) = \mathcal{L}_2(\rho)$ and $\mathcal{L}_2(\rho) = \mathcal{L}_2(\rho)$.

$$\alpha = \frac{\mathcal{L}_2(\rho)}{\mathcal{L}_2(\rho)} = 1 \text{ and } \mathcal{L}_2(\rho) = \mathcal{L}_2(\rho).$$

$$\mathcal{L}_2^1(\rho) = \mathcal{L}_2(\rho) \text{ and } \mathcal{L}_2^1(\rho) = \mathcal{L}_2(\rho).$$

$$\longrightarrow \mathcal{L}_2^1(\rho) = \mathcal{L}_2(\rho) \text{ and } \mathcal{L}_2^1(\rho) = \mathcal{L}_2(\rho).$$

Ex. 22 Use Eqs. (19-16) and (19-17)

$$\begin{aligned} \alpha_1' &= -\frac{1}{2} \frac{\partial^2}{\partial x^2} & \alpha_2' &= -\frac{\partial^2}{\partial x^2} \frac{\partial}{\partial x} \\ \beta_1(x,t) &= \beta_0 \left[\beta_0 \gamma_0 \frac{\partial^2}{\partial x^2} \right] \\ \beta_2(x,t) &= \beta_0 \left[\beta_0 \gamma_0 \frac{\partial^2}{\partial x^2} \right] \end{aligned}$$

Ex. 23

$$\begin{aligned} \alpha_1' &= \alpha_0 \frac{\partial}{\partial x} \beta_0 \gamma_0 & \alpha_2' &= \alpha_0 \frac{\partial}{\partial x} \beta_0 \gamma_0 \frac{\partial^2}{\partial x^2} \\ \beta_1' &= \frac{1}{2} \frac{\partial^2}{\partial x^2} \beta_0 \gamma_0 & \beta_2' &= \frac{1}{2} \frac{\partial^2}{\partial x^2} \beta_0 \gamma_0 \frac{\partial^2}{\partial x^2} \\ \beta_3' &= \frac{1}{2} \frac{\partial^2}{\partial x^2} \beta_0 \gamma_0 \frac{\partial^2}{\partial x^2} & \beta_4' &= \frac{1}{2} \frac{\partial^2}{\partial x^2} \beta_0 \gamma_0 \frac{\partial^2}{\partial x^2} \frac{\partial^2}{\partial x^2} \end{aligned}$$

Ex. 24

$$\begin{aligned} \alpha_1' &= \alpha_0 \frac{\partial}{\partial x} \beta_0 \gamma_0 \frac{\partial^2}{\partial x^2} & \alpha_2' &= \alpha_0 \frac{\partial}{\partial x} \beta_0 \gamma_0 \frac{\partial^2}{\partial x^2} \frac{\partial^2}{\partial x^2} \\ \beta_1' &= \frac{1}{2} \frac{\partial^2}{\partial x^2} \beta_0 \gamma_0 \frac{\partial^2}{\partial x^2} & \beta_2' &= \frac{1}{2} \frac{\partial^2}{\partial x^2} \beta_0 \gamma_0 \frac{\partial^2}{\partial x^2} \frac{\partial^2}{\partial x^2} \\ \beta_3' &= \frac{1}{2} \frac{\partial^2}{\partial x^2} \beta_0 \gamma_0 \frac{\partial^2}{\partial x^2} \frac{\partial^2}{\partial x^2} & \beta_4' &= \frac{1}{2} \frac{\partial^2}{\partial x^2} \beta_0 \gamma_0 \frac{\partial^2}{\partial x^2} \frac{\partial^2}{\partial x^2} \frac{\partial^2}{\partial x^2} \end{aligned}$$

Ex. 25

$$\begin{aligned} \alpha_1' &= \alpha_0 \frac{\partial}{\partial x} \beta_0 \gamma_0 \frac{\partial^2}{\partial x^2} & \alpha_2' &= \alpha_0 \frac{\partial}{\partial x} \beta_0 \gamma_0 \frac{\partial^2}{\partial x^2} \frac{\partial^2}{\partial x^2} \\ \beta_1' &= \frac{1}{2} \frac{\partial^2}{\partial x^2} \beta_0 \gamma_0 \frac{\partial^2}{\partial x^2} & \beta_2' &= \frac{1}{2} \frac{\partial^2}{\partial x^2} \beta_0 \gamma_0 \frac{\partial^2}{\partial x^2} \frac{\partial^2}{\partial x^2} \\ \beta_3' &= \frac{1}{2} \frac{\partial^2}{\partial x^2} \beta_0 \gamma_0 \frac{\partial^2}{\partial x^2} \frac{\partial^2}{\partial x^2} & \beta_4' &= \frac{1}{2} \frac{\partial^2}{\partial x^2} \beta_0 \gamma_0 \frac{\partial^2}{\partial x^2} \frac{\partial^2}{\partial x^2} \frac{\partial^2}{\partial x^2} \end{aligned}$$

$$\begin{aligned} \text{Setting } \gamma &= \alpha_0 \text{ in } \alpha_1' = \frac{\partial^2}{\partial x^2} \beta_0 \gamma_0 \text{ and} \\ &\text{in } \alpha_2' = \frac{\partial^2}{\partial x^2} \beta_0 \gamma_0 \frac{\partial^2}{\partial x^2} \end{aligned}$$

and equating, we obtain

$$\begin{aligned} \frac{\partial^2}{\partial x^2} \frac{\partial^2}{\partial x^2} \beta_0 \gamma_0 &= -\frac{\partial^2}{\partial x^2} \beta_0 \gamma_0 \\ \text{---} &\text{---} \frac{\partial^2}{\partial x^2} = -\frac{\partial^2}{\partial x^2} \gamma_0 \end{aligned}$$

Ex. 26 a) Sol. The odd and even TE modes are the propagating modes. Setting $\beta_0 = 1$ in the formulas in Table 19-1, we have

$$\beta_0 = \frac{1}{\sqrt{1 - \beta_0^2 \gamma_0^2}} \quad \text{for odd TE modes,}$$

$$\beta_0 = \frac{1}{\sqrt{1 - \beta_0^2 \gamma_0^2 - \gamma_0^2}} \quad \text{for even TE modes,}$$

4) Electric field inside — From E_{net} (Gauss's law):

$$\begin{aligned} E_{\text{net}} &= 0, \quad E_{\text{top}} + E_{\text{bot}} = -\frac{\rho}{\epsilon_0} \int_V dV \\ E_{\text{top}} + E_{\text{bot}} &= -\frac{\rho}{\epsilon_0} \int_V dV \end{aligned}$$

$$\begin{aligned} \text{Find charge density in conductor } E_{\text{top}} &= E_{\text{bot}} = E \Big|_{\text{top}} \\ E_{\text{top}} &= -E_{\text{bot}} \Rightarrow -E_{\text{bot}} = -\frac{\rho}{\epsilon_0} \int_V dV \end{aligned}$$

$$\begin{aligned} \text{Find charge density on conductor } E_{\text{top}} &= E_{\text{bot}} = E \Big|_{\text{top}} \\ E_{\text{top}} &= E_{\text{bot}} \Rightarrow E_{\text{top}} = -\frac{\rho}{\epsilon_0} \int_V dV \end{aligned}$$

For given E_{net} inside — From problem 4 (a-c):

$$\begin{aligned} E_{\text{top}} &= E_{\text{bot}}, \quad E_{\text{top}} + E_{\text{bot}} = -\frac{\rho}{\epsilon_0} \int_V dV \\ E_{\text{top}} + E_{\text{bot}} &= -\frac{\rho}{\epsilon_0} \int_V dV \\ E_{\text{top}} + E_{\text{bot}} &= E_{\text{top}} + E_{\text{bot}} \end{aligned}$$

$$\begin{aligned} \therefore E_{\text{top}} &= E_{\text{bot}} = \left(E_{\text{top}} + E_{\text{bot}} \right) = -\frac{\rho}{\epsilon_0} \int_V dV \\ E_{\text{top}} &= E_{\text{bot}} = -\frac{\rho}{\epsilon_0} \int_V dV \end{aligned}$$

Ex. 11.1 For a uniformly charged infinite slab, $E_{\text{top}} = E_{\text{bot}} = 0$

$$\begin{aligned} \text{a) For } x < -a: \quad E_{\text{top}} &= 0, \quad E_{\text{bot}} = E_{\text{top}} + E_{\text{bot}} \\ E_{\text{top}} &= 0, \quad E_{\text{bot}} = E_{\text{top}} + E_{\text{bot}} \\ E_{\text{top}} &= 0, \quad E_{\text{bot}} = E_{\text{top}} + E_{\text{bot}} \\ E_{\text{top}} &= 0, \quad E_{\text{bot}} = E_{\text{top}} + E_{\text{bot}} \end{aligned} \quad \begin{aligned} \text{b) For } -a < x < a: \quad E_{\text{top}} &= E_{\text{bot}} = E_{\text{top}} + E_{\text{bot}} \\ E_{\text{top}} &= E_{\text{bot}} = E_{\text{top}} + E_{\text{bot}} \\ E_{\text{top}} &= E_{\text{bot}} = E_{\text{top}} + E_{\text{bot}} \\ E_{\text{top}} &= E_{\text{bot}} = E_{\text{top}} + E_{\text{bot}} \end{aligned}$$

Similarly, for $x > a$:

$$\begin{aligned} E_{\text{top}} &= E_{\text{bot}} = E_{\text{top}} + E_{\text{bot}} \\ E_{\text{top}} &= E_{\text{bot}} = E_{\text{top}} + E_{\text{bot}} \end{aligned}$$

$$\text{c) Boundary conditions: } \begin{cases} \text{for } E_{\text{top}} & E_{\text{top}} = E_{\text{top}} + E_{\text{bot}} \\ \text{for } E_{\text{bot}} & E_{\text{bot}} = E_{\text{top}} + E_{\text{bot}} \end{cases}$$

$$\frac{E_{\text{top}}}{E_{\text{bot}}} \rightarrow \text{Characteristic equation for symmetric slab: } \frac{E_{\text{top}}}{E_{\text{bot}}} = -\frac{E_{\text{top}}}{E_{\text{bot}}}$$

Solve From Eq. (a)-(b): $R_{avg} = \frac{R}{2} \sqrt{\frac{2\pi}{12} + \frac{(2\pi)^2}{(12)^2 - 12}}$

$$R_{avg} = 0.000075 \text{ ft} (9 \text{ in}) \quad , \quad R_{avg} = \sqrt{\frac{2\pi}{12} + \frac{(2\pi)^2}{(12)^2 - 12}}$$

Calculate order modes and resonant frequencies:

Modes	R_{avg}	$(R_{avg}/a)^2$
$TM_{0,1}$	0.0001	2.11×10^8
$TE_{0,1}$	0.0001	1.136×10^8
$TM_{1,0}$	0.0002	1.666×10^8
$TE_{1,0}, TM_{0,2}$	0.0003	2.11×10^8
$TE_{1,1}$	0.0005	4.444×10^8
$TM_{2,0}$	0.0004	6.777×10^8
$TE_{2,0}, TM_{1,2}$	0.0006	1.111×10^9
$TE_{2,1}$	0.0008	1.666×10^9
$TM_{3,0}$	0.0009	2.111×10^9
$TE_{3,0}, TM_{2,2}$	0.0012	2.777×10^9

Ex. 12.11 a) From (a) and (b), the lowest-order resonant mode is $TE_{0,1}$ mode.

$$f_{0,1} = \frac{1}{2a} \sqrt{\frac{2\pi}{12} + \frac{\pi^2}{36}} = 4.444 \times 10^8 \text{ (Hz)}$$

b) From Eq. (a)-(b):

$$\begin{aligned} R_{avg} &= \frac{R_1 R_2 \sqrt{2\pi \ln(12) + \pi^2}}{2 \sqrt{2\pi \ln(12) + \pi^2} + \ln(12) \sqrt{2\pi}} \quad \left(R_1 = \sqrt{\frac{2\pi \ln(12)}{\pi}} \right) \\ &= \frac{0.0001 \times 0.0002 \sqrt{2\pi \ln(12) + \pi^2}}{2 \sqrt{2\pi \ln(12) + \pi^2} + \ln(12) \sqrt{2\pi}} = 0.0001 \end{aligned}$$

From Eq. (a)-(c) and (b)-(c):

$$R_1 = \frac{1}{2} R_2 \sqrt{\frac{2\pi}{12} + \frac{\pi^2}{36}} = 0.00012 \times 10^9 \text{ (Hz)}$$

$$R_2 = \frac{1}{2} \ln(12) \left(\frac{2\pi}{12} + \frac{\pi^2}{36} \right) R_1^2 = 0.00009 \times 10^9 \text{ (Hz)} = 90 \text{ Hz}$$

Ex. 20-20 (a) $Q_{\text{ext}} = \frac{7}{3} \sqrt{\frac{1}{2} + \frac{1}{2}} = \frac{7}{3} Q_{\text{ext}} = 1.07 \times 10^6 \text{ (W)}$
 (b) $Q_{\text{ext}} = \frac{1}{2} \rho_{\text{ext}} Q_{\text{ext}} = 1.07 \times 10^6$
 (c) $Q_{\text{ext}} = Q_{\text{ext}} = 1.07 \times 10^6 \text{ (W)} = 1.07 \times 10^6 \text{ (W)}$
 $= Q_{\text{ext}}$

Ex. 20-21 (a) Combining Eqs. (20-29) and (20-30)

$$Q_{\text{ext}} = \frac{1}{2} \rho_{\text{ext}} \frac{b (a^2 + d^2)^{3/2}}{[2b(a^2 + d^2) + d^2(a^2 + d^2)]}$$

→ Q_{ext} has a symmetrical dependence on a and d . It will be maximum when $a=d$, which gives a max. volume-to-surface ratio

(b) When $a=d$, $Q_{\text{ext}} = \frac{1}{2} \rho_{\text{ext}} \frac{b (1 + 1)^{3/2}}{2b(1 + 1) + d^2(1 + 1)}$

Ex. 20-22 (a) $Q_{\text{ext}} = \frac{1}{2} \rho_{\text{ext}} \frac{b (a^2 + d^2)^{3/2}}{2b(a^2 + d^2) + d^2(a^2 + d^2)}$

For $a=d=1.0$, $Q_{\text{ext}} = \frac{1}{2} \rho_{\text{ext}} \sqrt{\frac{1}{2} + \frac{1}{2}} = 1.07 \times 10^6 \text{ (W)}$

$Q_{\text{ext}} = 1.07 \times 10^6$

(b) For $Q_{\text{ext}} = 1.07 \times 10^6$, $1.07 \times 10^6 = 1.07 \times 10^6$

Ex. 20-23 (a) From the field configuration in the cavity we see that the Q_{ext} mode with respect to x is the same as the Q_{ext} mode with respect to y . Thus, $(Q_{\text{ext}})_{\text{ext}}$ can be obtained from $(Q_{\text{ext}})_{\text{ext}}$ in Eq. (20-30) by changing b to d and d to b .

(b) (i) & (ii) For the Q_{ext} mode can be derived from the field expressions in Eqs. (20-29), (20-30), & (20-31) by setting $m=n=1$, and using Eq. (20-32).

$Q_{\text{ext}} = 1.07 \times 10^6 \text{ (W)}$ or Q_{ext}

$Q_{\text{ext}} = \frac{1}{2} \rho_{\text{ext}} \frac{b (a^2 + d^2)^{3/2}}{2b(a^2 + d^2) + d^2(a^2 + d^2)}$

$$\begin{aligned}
 E_1 &= E_1 \left\{ \left(\int_0^1 (x_1 \sin \pi t) dt \right)^2 + \left(\int_0^1 (x_2 \sin \pi t) dt \right)^2 \right. \\
 &\quad \left. + \left(\int_0^1 (x_3 \sin \pi t + x_4 \sin \pi t) dt \right)^2 \right\} \\
 &= \frac{1}{4} \left(\frac{4}{\pi^2} \right) E \left\{ \frac{1}{2} \left(\frac{1}{2} \right)^2 + \frac{1}{2} \left(\frac{1}{2} \right)^2 + \frac{1}{2} \left(\frac{1}{2} \right)^2 \right\} \\
 &= \left(\frac{1}{2} \right)^2 + \left(\frac{1}{2} \right)^2
 \end{aligned}$$

$$E_{\text{out}} = \frac{E_1}{4} = \frac{2 \left(\frac{1}{2} \right)^2 + 2 \left(\frac{1}{2} \right)^2}{4 \left(\frac{1}{2} \right)^2 + 4 \left(\frac{1}{2} \right)^2 + 4 \left(\frac{1}{2} \right)^2} = E_1 \frac{1}{3} = \frac{2}{3}$$

Ex. 10.10 a) TM_{long} mode:

$$E_1^2 = E_0 E_2 \left(\frac{E_1}{E_0} \right) \cos \theta \cos \left(\frac{E_1}{E_0} \theta \right)$$

$$\theta = \alpha_1, \alpha_2, \dots, \theta = \beta_1, \beta_2, \dots, \theta = \beta_3, \beta_4, \dots$$

$$C_{\text{TM long}} = \frac{1}{\pi \left(\frac{E_1}{E_0} \right) \left(\left(\frac{E_1}{E_0} \right)^2 + \left(\frac{E_1}{E_0} \right)^2 \right)}$$

$$C_{\text{TM long}} = \frac{1}{\pi \left(\frac{E_1}{E_0} \right) \left(\left(\frac{E_1}{E_0} \right)^2 + \left(\frac{E_1}{E_0} \right)^2 \right)}$$

$$\text{TM}_{\text{long}} \text{ mode: } E_1^2 = E_0 E_2 \left(\frac{E_1}{E_0} \right) \cos \theta \cos \left(\frac{E_1}{E_0} \theta \right)$$

$$\theta = \alpha_1, \alpha_2, \dots, \theta = \beta_1, \beta_2, \dots, \theta = \beta_3, \beta_4, \dots$$

$$C_{\text{TM long}} = \frac{1}{\pi \left(\frac{E_1}{E_0} \right) \left(\left(\frac{E_1}{E_0} \right)^2 + \left(\frac{E_1}{E_0} \right)^2 \right)}$$

$$C_{\text{TM long}} = \frac{1}{\pi \left(\frac{E_1}{E_0} \right) \left(\left(\frac{E_1}{E_0} \right)^2 + \left(\frac{E_1}{E_0} \right)^2 \right)}$$

b) For $\alpha = 0$, the dominant mode is TM_{long}. $C_{\text{TM long}} = \frac{1}{\pi \left(\frac{E_1}{E_0} \right) \left(\left(\frac{E_1}{E_0} \right)^2 + \left(\frac{E_1}{E_0} \right)^2 \right)}$

The first seven modes with the lowest resonant frequency

Mode	TM _{long}	TE _{long}	TM _{short}	TM _{long}	TE _{long}	TM _{short}	TE _{long}
E/E_0	1.00	1.17	1.17	1.43	1.43	1.66	1.76

Ex. 10.11 $C = \frac{E_1}{E_0} = \frac{E_1}{E_0} \cos \left(\frac{E_1}{E_0} \right)$ $L = \frac{E_1}{E_0} \sin \left(\frac{E_1}{E_0} \right)$

a) $L_0 = \frac{1}{\pi \left(\frac{E_1}{E_0} \right)} = \frac{1}{\pi \left(\frac{E_1}{E_0} \right) \left(\left(\frac{E_1}{E_0} \right)^2 + \left(\frac{E_1}{E_0} \right)^2 \right)}$

b) $L_0 = \frac{1}{\pi \left(\frac{E_1}{E_0} \right)} = \frac{1}{\pi \left(\frac{E_1}{E_0} \right) \left(\left(\frac{E_1}{E_0} \right)^2 + \left(\frac{E_1}{E_0} \right)^2 \right)}$

Chapter II

Antennas and Radiating Systems

Ex. 1 Maxwell's equations for simple media:

$$\nabla \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t}, \quad (1)$$

$$\nabla \times \vec{H} = \vec{J} + \epsilon \frac{\partial \vec{E}}{\partial t}, \quad (2)$$

$$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon}, \quad (3)$$

$$\nabla \cdot \vec{H} = 0. \quad (4)$$

$$\begin{aligned} \text{a) } \nabla \times \vec{E} &= \nabla \times (\nabla \times \vec{E}) = \nabla(\nabla \cdot \vec{E}) - \nabla^2 \vec{E} \\ &= -\mu \frac{\partial \vec{H}}{\partial t} - \nabla^2 \vec{E}. \end{aligned} \quad (5)$$

$$\begin{aligned} \text{b) } \nabla \times (\nabla \times \vec{E}) &= \nabla(\nabla \cdot \vec{E}) - \nabla^2 \vec{E} \\ &= \frac{1}{\epsilon} \nabla \rho - \nabla^2 \vec{E}. \end{aligned} \quad (6)$$

Combining (5) and (6), we obtain

$$\nabla^2 \vec{E} - \mu \epsilon \frac{\partial^2 \vec{E}}{\partial t^2} = -\frac{1}{\epsilon} \nabla \rho + \mu \frac{\partial \vec{J}}{\partial t}.$$

$$\text{c) Similarly, we have } \nabla^2 \vec{H} - \mu \epsilon \frac{\partial^2 \vec{H}}{\partial t^2} = -\nabla \times \vec{J}.$$

Ex. 2 $\vec{E}_y(x, y, z) = -\nabla \psi = -\hat{x} \psi_x - \hat{y} \psi_y - \hat{z} \psi_z.$

$$E_x = -\frac{\partial \psi}{\partial x} = j \omega A_x \quad \text{The expression of } A_x, A_y,$$

$$E_y = -\frac{\partial \psi}{\partial y} = j \omega A_y \quad \text{and } A_z \text{ are given in Ex. 1}$$

$$E_z = -\frac{\partial \psi}{\partial z} = j \omega A_z \quad (\text{in this case}),$$



$$\psi = \frac{A_0}{4\pi\epsilon_0} \left[\frac{e^{j\omega t - \beta r}}{r} - \frac{e^{j\omega t - \beta r'}}{r'} \right],$$

$$A_x = A - j \omega \cos \theta,$$

$$A_y = A - j \omega \sin \theta,$$

$$A = \frac{I_0}{j\omega}, \quad \text{with } \omega < \beta c.$$

$$\begin{aligned} \psi &= \frac{A_0 e^{j\omega t}}{4\pi\epsilon_0} \frac{1}{\beta} \int_{-\beta z'}^{+\beta z} (1 - \beta \cos \theta) e^{j\omega t - \beta r} - (1 - \beta \cos \theta') e^{j\omega t - \beta r'} \\ &= \frac{A_0 e^{j\omega t}}{4\pi\epsilon_0 \beta} \left[(1 - \beta \cos \theta) \left(\frac{e^{j\omega t - \beta r}}{r} \right) - (1 - \beta \cos \theta') \left(\frac{e^{j\omega t - \beta r'}}{r'} \right) \right] \end{aligned}$$

$$\begin{aligned}
 v &= \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \left(1 + \frac{1}{c^2} \frac{d^2 x}{dt^2} \right) dx \\
 &= \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \left(1 + \frac{1}{c^2} \frac{d^2 x}{dt^2} \right) v dt
 \end{aligned}$$

Using A_1, A_2, A_3 , and v in A_1, A_2 , and A_3 , we obtain the same results as given in Eqs. (17a), (17b), and (17c).

Case 1



$$\begin{aligned}
 A_1 &= \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \int \frac{d^2 x}{dt^2} dx \\
 &= \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \int \frac{d^2 x}{dt^2} v dt
 \end{aligned}$$

$$\begin{aligned}
 A_1 &= \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \left(1 + \frac{1}{c^2} \frac{d^2 x}{dt^2} \right) \int v dt \\
 &= \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \int v dt
 \end{aligned}$$

$$\int v dt = 0$$

$$\int \frac{d^2 x}{dt^2} dx = \int \frac{d^2 x}{dt^2} v dt = \int \frac{d^2 x}{dt^2} v dt = \int \frac{d^2 x}{dt^2} v dt$$

$$A_1 = 0 + 0 + 0 + 0 = 0$$

$$A_2 = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \int \frac{d^2 y}{dt^2} dy = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \int \frac{d^2 y}{dt^2} v dt = 0$$

$$A_3 = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \int \frac{d^2 z}{dt^2} dz = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \int \frac{d^2 z}{dt^2} v dt = 0$$

$$\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \left(1 + \frac{1}{c^2} \frac{d^2 x}{dt^2} \right) \int v dt = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \left(1 + \frac{1}{c^2} \frac{d^2 x}{dt^2} \right) \int v dt = 0$$

$$\begin{aligned}
 \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \left(1 + \frac{1}{c^2} \frac{d^2 x}{dt^2} \right) \int v dt &= \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \left(1 + \frac{1}{c^2} \frac{d^2 x}{dt^2} \right) \int v dt = 0 \\
 &= \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \left(1 + \frac{1}{c^2} \frac{d^2 x}{dt^2} \right) \int v dt = 0
 \end{aligned}$$

In the same manner, we obtain

$$\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \left(1 + \frac{1}{c^2} \frac{d^2 y}{dt^2} \right) \int v dt = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \left(1 + \frac{1}{c^2} \frac{d^2 y}{dt^2} \right) \int v dt = 0$$

$$\therefore \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \left(1 + \frac{1}{c^2} \frac{d^2 z}{dt^2} \right) \int v dt = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \left(1 + \frac{1}{c^2} \frac{d^2 z}{dt^2} \right) \int v dt = 0$$

and

$$\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \left(1 + \frac{1}{c^2} \frac{d^2 x}{dt^2} \right) \int v dt = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \left(1 + \frac{1}{c^2} \frac{d^2 x}{dt^2} \right) \int v dt = 0$$

Let $m = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$

$$A_1 = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \left(1 + \frac{1}{c^2} \frac{d^2 x}{dt^2} \right) \int v dt = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \left(1 + \frac{1}{c^2} \frac{d^2 x}{dt^2} \right) \int v dt = 0$$

$$= \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \left(1 + \frac{1}{c^2} \frac{d^2 x}{dt^2} \right) \int v dt = 0$$

$$\begin{aligned} \text{a) } \vec{E} &= \frac{1}{\epsilon_0} \nabla \times \vec{F} = E_x \hat{x} + E_y \hat{y} && \text{Expression for } E_x, E_y \\ &&& \text{and } E_z \text{ same as those} \\ \text{b) } \vec{F} &= \frac{1}{\mu_0} \nabla \times \vec{E} = F_x \hat{x} + F_y \hat{y} && \text{given in Eq. (2) and (3)} \end{aligned}$$

In the far zone, $\rho \rightarrow \infty$, $\frac{1}{\rho} \frac{\partial}{\partial \rho}$ and $\frac{1}{\rho^2} \frac{\partial}{\partial \rho}$ terms can be neglected. We have the following instantaneous expressions assuming spherical waves:

$$\begin{aligned} E_x(\rho, z) &= -E_0 \frac{e^{i(k\rho - \omega t)}}{\rho} \sin \theta \cos(\omega t - \phi_0) \\ E_y(\rho, z) &= E_0 \frac{e^{i(k\rho - \omega t)}}{\rho} \sin \theta \sin(\omega t - \phi_0) \\ F_x(\rho, z) &= -E_0 \frac{e^{i(k\rho - \omega t)}}{c\rho} \sin \theta \cos(\omega t - \phi_0) \end{aligned}$$

Ex-2 Far-zone electric field of elemental electric dipole

$$E_x(\rho, z) = \frac{E_0}{\rho} \left(\frac{z^2}{\rho^2} \right) \sin \theta \cos(\omega t - \phi_0) = E_0 \sin \theta \cos(\omega t - \phi_0) \frac{z^2}{\rho^3}$$

For the elemental magnetic dipole:

$$E_y(\rho, z) = \frac{E_0}{\rho} \left(\frac{z^2}{\rho^2} \right) \sin \theta \sin(\omega t - \phi_0) = E_0 \sin \theta \sin(\omega t - \phi_0) \frac{z^2}{\rho^3}$$

$$\text{a) Then, } \frac{\left(\frac{E_x(\rho, z)}{E_0} \right) \cos \theta}{\left(\frac{E_y(\rho, z)}{E_0} \right) \sin \theta} = \frac{\left(\frac{z^2}{\rho^3} \right) \cos \theta}{\left(\frac{z^2}{\rho^3} \right) \sin \theta} = 1$$

————— Linear polarization

b) Circular polarization if $\theta = \pi/4$.

$$\text{Ex-3} \quad \text{Equation of continuity: } \nabla \cdot \vec{J} = -\frac{\partial \rho}{\partial t} \longrightarrow E_z = \frac{E_0}{\rho} \frac{\partial \rho}{\partial t}$$

$$\text{a) } E_{\theta} = E_0 \cos \theta \longrightarrow E_z = -j \frac{E_0}{\rho} E_{\theta} \sin \theta = -j \frac{E_0^2}{\rho} \sin \theta \cos \theta$$

$$\text{b) } E_{\phi} = E_0 \left(\sin \theta - \frac{z}{\rho} \cos \theta \right) \longrightarrow E_z = \left(-j \frac{E_0^2}{\rho} \sin \theta \cos \theta \right)$$

$$\text{Ex-4} \quad \lambda = \frac{2\pi}{k} \text{ where } k = \frac{\omega}{c} = \frac{2\pi \nu}{c} \text{ (Maxwell's eqn.)}$$

$$\text{a) Radiation resistance, } R_r = 80 \pi^2 \left(\frac{L}{\lambda} \right)^2 = 1.13 \times 10^{-3} \Omega$$

$$\text{b) } R_p(\omega = 0) = R_0 = \sqrt{\frac{2\pi \times 10^{-7} \times 10^{-12}}{1.13 \times 10^{-3}}} = 1.13 \times 10^{-4} \Omega$$

$$R_p(\omega = 0) = R_0 = R_1 \left(\frac{L}{\lambda_0} \right) = 1.13 \times 10^{-4} \Omega \longrightarrow R_1 = \frac{R_0}{L} = 10^{-12} \Omega$$

$$\text{c) } R_p(\omega = 0) = R_1 = \frac{R_0}{L} = 10^{-12} \Omega$$

$$R_p(\omega = 0) \left(\frac{L}{\lambda_0} \right) = \left(\frac{L}{\lambda_0} \right)^2 \frac{R_0}{L} \longrightarrow R_{\text{loss}} = \frac{1}{2} \frac{R_0}{L} = 10^{-13} \Omega$$

Ex. 11-22 a) $E_1 = \cos \theta \left(\frac{d^2}{dt^2} \right)^2 = 2\omega^2 \cos \theta \sin \theta \cos \theta$

b) $E_2 = \frac{d^2}{dt^2} E_1$, $E_3 = \frac{d^2}{dt^2} E_2$

c) $F_{\text{net}} = F = r \cos \theta \hat{r}$, $L_y = L_y = r \sin \theta \hat{\phi}$ (not in radial dir)
 $\hat{r} = \sin \theta \cos \phi \hat{x} + \sin \theta \sin \phi \hat{y} + \cos \theta \hat{z}$, $E_1 = L_y \cdot \hat{r} = r \sin^2 \theta \cos \phi$
 $E_2 = L_y \cdot \hat{r} = r \sin^2 \theta \cos \phi$, $E_3 = \frac{d^2}{dt^2} E_1$
 $\rightarrow E_3 = 2 \cos \theta \hat{z}$

Ex. 11-23 $E_1 = F \cdot \hat{r} = F \left(\frac{d^2}{dt^2} \right)^2 = \frac{1}{2} F \hat{r} \cdot \hat{r} = \frac{1}{2} F \hat{r} \cdot \hat{r}$
 $= \frac{1}{2} F \hat{r} \cdot \hat{r} = \frac{1}{2} F \hat{r} \cdot \hat{r} = \frac{1}{2} F \hat{r} \cdot \hat{r}$
 $= \frac{1}{2} F \left(\frac{d^2}{dt^2} \right)^2$, which is the same as Eq. (11-42)

Ex. 11-24 $F_{\text{net}} = \cos \theta \hat{r} \cdot \cos \theta \hat{r}$



For the vector,

$$F_{\text{net}} = \left| \frac{\cos \theta \cos \theta}{\cos \theta} \right|$$

Which of magnitude between
the first and
 $\cos \theta \cos \theta = \cos \theta \cos \theta$

Ex. 11-25 $F_{\text{net}} = L_y \left(1 - \frac{1}{2} \right)$

From Eq. (11-42), $E_1 = \frac{d^2}{dt^2} L_y \left(1 - \frac{1}{2} \right) \cos \theta \cos \theta$
 $= \frac{d^2}{dt^2} L_y \left(1 - \frac{1}{2} \right) \cos \theta \cos \theta$
 $= \frac{d^2}{dt^2} L_y \left(1 - \cos \theta \cos \theta \right)$

Maximum E_1 occurs at $\theta = \frac{\pi}{2}$, where
 $E_1 \left(\frac{\pi}{2} \right) = 1 - \frac{1}{2}$

Ex 11-10 a) $R_{\theta} = -R_{\theta} = -\frac{d}{dt} \left[\frac{\cos(\frac{2}{3}\pi) \sin(\frac{2}{3}\pi)}{\frac{2}{3}\pi} \right]$

b) $R_{\theta} = \frac{1}{\pi} \left[\frac{d}{dt} \sin \right] R_{\theta} = \frac{1}{\pi} \frac{d}{dt} \left[\frac{\cos(\frac{2}{3}\pi) \sin(\frac{2}{3}\pi)}{\frac{2}{3}\pi} \right]$

which has a maximum value $\frac{1}{\pi} \left(\frac{d}{dt} \sin \right) R_{\theta}$ at $\theta = \frac{\pi}{2}$

c) $R_{\theta} = \frac{1}{\pi} \frac{d}{dt} \sin = \frac{1}{\pi} \cos(\theta) = \frac{1}{\pi} \cos(\frac{\pi}{2}) = 0$ and $R_{\theta} = \frac{1}{\pi} \cos(\theta) = \frac{1}{\pi} \cos(\frac{\pi}{2}) = 0$

$\theta = \frac{\pi}{2} \Rightarrow R_{\theta}(\frac{\pi}{2}) = \frac{1}{\pi} \cos(\frac{\pi}{2}) = 0$ and $R_{\theta} = \frac{1}{\pi} \cos(\theta) = \frac{1}{\pi} \cos(\frac{\pi}{2}) = 0$

$R_{\theta} = \frac{1}{\pi} \frac{d}{dt} \sin = \frac{1}{\pi} \cos(\theta) = \frac{1}{\pi} \cos(\frac{\pi}{2}) = 0$

$\theta = \frac{\pi}{2} \Rightarrow R_{\theta}(\frac{\pi}{2}) = \frac{1}{\pi} \left[\frac{\cos(\frac{2}{3}\pi) \sin(\frac{2}{3}\pi)}{\frac{2}{3}\pi} \right] = \frac{1}{\pi} \cos(\theta)$

$R_{\theta} = \frac{1}{\pi} \cos(\theta)$ and $R_{\theta} = \frac{1}{\pi} \cos(\theta)$

Ex 11-11



$R_{\theta} = \frac{1}{\pi} \frac{d}{dt} \sin$

and $R_{\theta} = \frac{1}{\pi} \frac{d}{dt} \sin = \frac{1}{\pi} \cos(\theta)$

$R_{\theta} = \frac{1}{\pi} \cos(\theta)$

$R_{\theta} = \frac{1}{\pi} \cos(\theta) = \frac{1}{\pi} \cos(\frac{\pi}{2}) = 0$

where $R_{\theta} = \frac{1}{\pi} \cos(\theta) = \frac{1}{\pi} \cos(\frac{\pi}{2}) = 0$

a) $R_{\theta} = \frac{1}{\pi} \cos(\theta)$

$R_{\theta} = \frac{1}{\pi} \cos(\theta) = \frac{1}{\pi} \cos(\frac{\pi}{2}) = 0$



c) $R_{\theta} = \frac{1}{\pi} \cos(\theta)$

$R_{\theta} = \frac{1}{\pi} \cos(\theta) = \frac{1}{\pi} \cos(\frac{\pi}{2}) = 0$



Ex-10



From Ex-9 we have

$$z_1 = \frac{1}{\sqrt{2}} \left(\frac{1}{\sqrt{2}} + j \frac{1}{\sqrt{2}} \right) = \frac{1}{2} (1 + j)$$

$$z_2 = \frac{1}{\sqrt{2}} \left(\frac{1}{\sqrt{2}} - j \frac{1}{\sqrt{2}} \right) = \frac{1}{2} (1 - j)$$

where

$$|z_1| = |z_2|$$

$$= \left| \frac{1}{\sqrt{2}} \cos \alpha + j \frac{1}{\sqrt{2}} \sin \alpha \right|$$

$$= \left| \frac{1}{\sqrt{2}} \cos \alpha + j \frac{1}{\sqrt{2}} \sin \alpha \right|$$

$$= \sqrt{\frac{1}{2} + \frac{1}{2}} = 1$$

$$z_1 + z_2 = \frac{1}{2} (1 + j) + \frac{1}{2} (1 - j) = \frac{1}{2} (1 + 1 + j - j) = 1$$

From Ex-9 we have $|z_1| = |z_2| = 1$

a) In the complex plane: $\alpha = 45^\circ$, $z_1 = \frac{1}{\sqrt{2}} (1 + j)$

b) In the complex plane: $\alpha = 315^\circ$, $z_2 = \frac{1}{\sqrt{2}} (1 - j)$

c) In the complex plane: $\alpha = 45^\circ$, $z_1 = \frac{1}{\sqrt{2}} (1 + j)$

d) $\alpha = 45^\circ$, $z_1 = \frac{1}{\sqrt{2}} (1 + j)$



$$z_1 = \frac{1}{\sqrt{2}} (1 + j)$$



$$z_2 = \frac{1}{\sqrt{2}} (1 - j)$$

Ex 2.1 From Eq. (2.111) $M(\psi) = \frac{d^2}{d\psi^2} [\psi \cos(\psi)]$, where

$$\psi = \beta \sin(\alpha \sin \theta) \quad (2.112)$$

In the β -plane of a dipole, $\alpha = \pi/2$, $M(\beta) = \beta \cos \beta$,
 $\psi = \beta$, $\beta = \psi$.

$$M(\psi) = \beta \cos(\beta) = \psi \cos(\psi)$$



$$\text{In } \alpha = \pi/2, \quad \psi = \beta$$

$$M(\psi) = \beta \cos(\beta) = \psi \cos(\psi)$$



Ex 2.2 a) Relative radiation amplitudes: $F(\theta) = M(\beta)$.

b) Array factor: $M(\psi) = [\cos(\frac{\psi}{2}) \cos(\frac{\psi}{2})]^2$.



$$\text{a) } \cos(\frac{\psi}{2}) \cos(\frac{\psi}{2}) = (\cos \frac{\psi}{2})^2$$

$$\psi = \pi/2, \alpha = \pi/2$$

$$\text{In } \alpha = \pi/2 \text{ power beamwidth}$$

$$= 2 (\pi/2 - \pi/2) = \pi/2$$

$$= 90^\circ$$

For uniform array, from Eq. (2.111):

$$\frac{1}{\beta} \left(\frac{\sin \frac{\beta}{2}}{\frac{\beta}{2}} \right) = \frac{1}{\beta} \Rightarrow \beta = \pi/2$$

Half-power beamwidth for 3-element uniform array
 with $\alpha = \pi/2$ spacing $= 2 (\pi/2 - \pi/2) = \pi/2$.

Ex 2.3 a) From Eq. (2.111) the Array factor: $M(\psi) = \frac{d^2}{d\psi^2} [\psi \cos(\psi)]$



1) Resonance Operation $\phi = \phi_{\text{res}} = 0$

$$| \text{loop} | = \frac{1}{\beta} \left| \frac{A \beta V_{\text{TH}}}{1 + A \beta V_{\text{TH}}} \right| = \left| \frac{A \beta V_{\text{TH}}}{1} \right| \quad \text{for } \phi = 0$$

$$\text{where } X = A \beta V_{\text{TH}}$$

$$\text{At half-power points: } \left| \frac{A \beta V_{\text{TH}}}{1} \right| = \frac{1}{\sqrt{2}} \implies X = 0.707$$

(For both bandwidth and frequency)

For bandwidth operation, the half-power bandwidth

$$\therefore (A \beta V_{\text{TH}})_{\text{BW}} = 0.707 \left(\frac{A \beta V_{\text{TH}}}{1} \right) \text{ (max)}$$

$$= 0.707 \left(\frac{A \beta V_{\text{TH}}}{1} \right) \text{ (max)}$$

$$\text{For } A \beta V_{\text{TH}} = 0.707 \left(\frac{A \beta V_{\text{TH}}}{1} \right) \text{ (max)}$$

$$\text{From Eq. (10-55): } (A \beta V_{\text{TH}})_{\text{BW}} = 0.707 \left(\frac{A \beta V_{\text{TH}}}{1} \right) \text{ (max)}$$

2) Resonance Operation $\phi = \phi_{\text{res}} = 0$

$$(A \beta V_{\text{TH}})_{\text{BW}} = 0.707 \left(\frac{A \beta V_{\text{TH}}}{1} \right) \text{ (max)} = 0.707 \left(\frac{A \beta V_{\text{TH}}}{1} \right) \text{ (max)}$$

$$\text{For } A \beta V_{\text{TH}} = 0.707 \left(\frac{A \beta V_{\text{TH}}}{1} \right) \text{ (max)}$$

$$\text{From Eq. (10-55): } (A \beta V_{\text{TH}})_{\text{BW}} = 0.707 \left(\frac{A \beta V_{\text{TH}}}{1} \right) \text{ (max)}$$

Example: $| \text{loop} | = \frac{1}{\beta} \left| \frac{A \beta V_{\text{TH}}}{1 + A \beta V_{\text{TH}}} \right| = \left| \frac{A \beta V_{\text{TH}}}{1} \right|$, where $X = \frac{A \beta V_{\text{TH}}}{1}$

Resonance operation: $\phi = \phi_{\text{res}} = 0$

$$\beta = \frac{A \beta V_{\text{TH}}}{\sqrt{1 + (A \beta V_{\text{TH}})^2}}$$

$$\int_0^{\infty} \left(\frac{A \beta V_{\text{TH}}}{1} \right)^2 \sin \theta \, d\theta = \frac{A \beta V_{\text{TH}}}{\sqrt{1 + (A \beta V_{\text{TH}})^2}} \left(\frac{A \beta V_{\text{TH}}}{1} \right) = \frac{A \beta V_{\text{TH}}}{1}$$

$$\therefore \beta = \frac{A \beta V_{\text{TH}}}{1} = \frac{A \beta V_{\text{TH}}}{1}$$

Example: Construction follows the steps outlined in pp. 10-55



$$\beta_0 \text{ is at } \beta = \frac{A \beta V_{\text{TH}}}{1}$$

$$\text{Radius of circle is } \frac{A \beta V_{\text{TH}}}{1}$$

Ex-22 a) $A(x) = 1 + x$, $x = e^{i\theta}$ ψ at π

b) $A(x) = (1+x)^2$ A double zero $\psi_0 = \psi_2 = 0$

c) $A(x) = \frac{x^2-1}{x+1}$ Zero at ψ_0 at $\frac{\pi}{2}$, $\pi = 1, 3, 5, \dots$

d) $A(x) = (1+x+x^2) = \left(\frac{x^3-1}{x-1}\right)$

Double zero ($\psi_0 = \psi_2 = 1, 2, \dots$)

($\psi_1 = \psi_4 = 1, 3, 5, \dots$)

- e) The zeros of an array polynomial specify the nulls in the array pattern as θ changes from 0 to 2π . If ψ ($\psi = 0$) gives the main beam. The regions between the nulls (except the main beam region) are sidelobe. The double-zero at the array is partially or more widely spaced leading to a wider beamwidth and lower sidelobe levels. In fact, are 3 dB lower than those of a three-element uniform array.

Ex-23 Formula (1)-(3) and (5)-(6)

$$|E_s| = \frac{1}{\sqrt{N}} \left| \sum_{n=0}^{N-1} e^{j\psi_n} \right| \left| \sum_{m=0}^{N-1} A_m(x) e^{j\psi_m} \right|$$

where $A_m(x) = \frac{1}{N} \frac{\sin(N\psi_m)}{\sin(\psi_m)}$, $\psi_m = \frac{2\pi}{N} m \cos \theta$

$A_n(x) = \frac{1}{N} \frac{\sin(N\psi_n)}{\sin(\psi_n)}$, $\psi_n = \frac{2\pi}{N} n \cos \theta$

$$|E_s(x)| = \frac{1}{\sqrt{N}} \left| \left[\sum_{n=0}^{N-1} e^{j\psi_n} \right] \sum_{m=0}^{N-1} \left[\frac{\sin(N\psi_m)}{\sin(\psi_m)} \right] \right|$$

Ex-24 From Eq. (1) and (5) $G_p(x) = \frac{\sin(N\psi)}{\sin(\psi)} = \frac{\sin(N\psi)}{\sin(\psi)}$ (1)

Using Eq. (2) & (6) & (1) $G_p(x) = \cos \left[\frac{\sin(N\psi)}{\sin(\psi)} \right]$ (2)

d) Substituting (2) in Eq. (3) $A_p(x) = \frac{1}{N} G_p(x) = \cos \left[\frac{\sin(N\psi)}{\sin(\psi)} \right]$

e) When value of $A_p(0)$ & $A_p(\pi) = 1$ and

$F_n = \frac{1}{N} \sin^2(N\psi)$, $n = 1, 2, 3, \dots$, $A_p(\pi/2) = 0$

Ex. 11.21 a) $E_p(r) = \frac{1}{4\pi\epsilon_0} E_p = \frac{1}{4\pi\epsilon_0} \left(\frac{q}{r^2} \right) \left(\frac{4\pi}{3} r^3 \right) \epsilon_0 \text{ since}$

$$E_p = \int E_p \cdot d\vec{r} = \int_0^r \frac{q}{r'^2} \left(\frac{4\pi}{3} r'^3 \right) \epsilon_0 dr' \\ = \frac{4\pi}{3} q \epsilon_0 \left(\frac{r^3}{3} - \frac{r_0^3}{3} \right) \left(\frac{1}{r^2} \right) \quad (a)$$

$$\text{so } E_p = \frac{q}{3r} = \frac{4\pi}{3} q \epsilon_0 \left(\frac{r^3}{3} - \frac{r_0^3}{3} \right) \left(\frac{1}{r^2} \right) = \frac{1}{3} E_0$$

Ex. 11.22 From Ex. 11.21 (a) $\frac{q}{3r} = \left(\frac{4\pi}{3} \epsilon_0 \right) E_p \epsilon_0$

a) For half-crown dielectric: $E_p = E_0 = 1.00$

$$r = \frac{q}{3E_p} \left(\frac{4\pi}{3} \epsilon_0 \right) \quad \text{so } q = 3E_p r \left(\frac{3}{4\pi} \right) \left(\frac{1}{\epsilon_0} \right) \quad r = 0.5 \times 10^{-2} \text{ (m)}$$

$$E_p = 2.00 \times 10^6 \text{ V/m} \quad q = 0.47 \times 10^{-6} \text{ (C)} = 0.47 \text{ } \mu\text{C (approx.)}$$

b) For Krypton dielectric: $E_p = E_0 = 1.00$

$$r_p = 0.47 \text{ } \mu\text{C (approx.)}$$

Ex. 11.23 From Ex. 11.22, Earth radius = 6370 (km)

Geosynchronous orbit radius = 42,164 (km)



$$\psi = \sin^{-1} \left(\frac{6370}{42164} \right) = 8.74^\circ \text{ (approx.)}$$

$$\psi' = 90^\circ - \psi = 81.2^\circ$$

a) Two satellites would never meet

$$\text{so } (2\psi')^2 = (282.4^\circ)^2 < 360^\circ$$

Use three satellites in this
geostationary plane

$$3 = (2\psi')^2 = 282.4^\circ < 360^\circ$$

$\psi' < 90^\circ$ — cannot happen in
polar regions

b) Let E_0 = Power transmitted by satellite antenna

$$E_0 = \text{Power density within the cone} = \frac{E_0}{4\pi r^2} E_0$$

$$\text{Area of cone cap on earth} = \int_0^{\psi'} \int_0^{2\pi} r^2 \sin \theta d\theta d\phi \\ = 2\pi r^2 \int_0^{\psi'} \sin \theta d\theta = 2\pi r^2 (1 - \cos \psi') = 2\pi r^2$$

$$\therefore E_0 = 2\pi r^2 E_0 \text{ — so } \psi = \frac{1}{2} \sqrt{\frac{E_0}{E_0}} = \frac{1}{2} \sqrt{\frac{E_0}{E_0}}$$

$$\text{Satellite-to-Earth radius} = 2r = \frac{1}{2} \sqrt{\frac{E_0}{E_0}}$$

Ex-12 a) From Eq (10-102) $A_2 = \frac{2\pi\epsilon_0\epsilon_0}{\epsilon_0} E_1$.

$$A_2 = \frac{2\pi\epsilon_0\epsilon_0}{\epsilon_0} E_1 = 2.54 \times 10^{-10} \text{ C/m}, \quad Q_2 = 2\pi\epsilon_0\epsilon_0 E_1 = 2.54 \times 10^{-10} \text{ C/m}$$

$$A_3 = \frac{2\pi\epsilon_0\epsilon_0}{\epsilon_0} E_1 = 2.54 \times 10^{-10} \text{ C/m}, \quad Q_3 = 2\pi\epsilon_0\epsilon_0 E_1 = 2.54 \times 10^{-10} \text{ C/m}$$

$$F = 2.54 \times 10^{-10} \text{ C/m}, \quad E_1 = 2.54 \times 10^{-10} \text{ C/m}$$

$$\rightarrow A_2 = 2.54 \text{ C/m}$$

b) From Eq (10-102) $A_2 = \frac{2\pi\epsilon_0\epsilon_0}{\epsilon_0} \left(\frac{2\pi\epsilon_0\epsilon_0}{\epsilon_0} \right) E_1$.

$$A_2 = \frac{2\pi\epsilon_0\epsilon_0}{\epsilon_0} E_1 = 2.54 \times 10^{-10} \text{ C/m}, \rightarrow A_2 = 2.54 \text{ C/m}$$

Ex-13 a) From Eq (10-102) $A_2 = \frac{2\pi\epsilon_0\epsilon_0}{\epsilon_0} \left(\frac{2\pi\epsilon_0\epsilon_0}{\epsilon_0} \right) E_1$.

$$\text{where, From Eq (10-102), } E_1 = \frac{1}{4\pi\epsilon_0\epsilon_0} \left(\frac{2\pi\epsilon_0\epsilon_0}{\epsilon_0} \right) E_1$$

$$= 2.54 \times 10^{-10} \text{ C/m}$$

$$\text{Using Eq (10-102) } E_1 = 2.54 \times 10^{-10} \text{ C/m} = 2.54 \times 10^{-10} \text{ C/m}$$

b) From Eq (10-102) $\frac{2\pi\epsilon_0\epsilon_0}{\epsilon_0} = 2.54 \times 10^{-10} \text{ C/m}$

$$\rightarrow A_2 = 2.54 \text{ C/m}$$

Ex-14



$$E_1 = \frac{q}{4\pi\epsilon_0\epsilon_0 r^2}$$

$$\text{At } E_2 = \frac{q}{4\pi\epsilon_0\epsilon_0 r^2} \int_0^L \frac{2\pi\epsilon_0\epsilon_0}{\epsilon_0} E_1 \cos\theta \, dx$$

$$\text{In the above case,}$$

$$E_1 = 2.54 \times 10^{-10} \text{ C/m}$$

$$E_2 = \frac{q}{4\pi\epsilon_0\epsilon_0 r^2} \int_0^L \frac{2\pi\epsilon_0\epsilon_0}{\epsilon_0} E_1 \cos\theta \, dx$$

$$= \frac{q}{4\pi\epsilon_0\epsilon_0 r^2} \int_0^L \frac{2\pi\epsilon_0\epsilon_0}{\epsilon_0} E_1 \cos\theta \, dx$$

$$\text{where } E_1 = \frac{q}{4\pi\epsilon_0\epsilon_0 r^2}$$

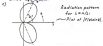
a) $A_2 = A_1 \cos\theta$, $A_3 = A_1 \sin\theta$, $A_4 = 0$

$$R = \frac{1}{2} R_0 \cos \theta = R_0 \frac{1}{2} \left(\frac{1}{2} \cos 2\theta + \frac{3}{2} \cos^3 \theta \right).$$

In the x - y plane, $\frac{1}{2} \frac{R^2}{R_0^2} = \frac{1}{2} \implies R = R_0 \frac{1}{2} \frac{1}{2} \cos 2\theta$

$$R(x, y) = R_0 \frac{1}{2} \frac{1}{2} \cos 2\theta = \frac{R_0}{4} \cos 2\theta = \frac{R_0}{4} \left(\frac{x^2 - y^2}{R^2} \right).$$

$$R(x, y) = R_0 \frac{1}{4} \frac{x^2 - y^2}{R^2}.$$



Ex-22 From Eq. (12-74) $E_z = \frac{1}{2} (E_0 + E_0 e^{i\theta})$

a) $E_z = E_0 (E_0 + E_0 e^{i\theta}) e^{i\theta}$

From Eq. (12-74) $|E_z| = |E_0 + E_0 e^{i\theta}| = \frac{1}{2} (1 + e^{i\theta}) = \frac{1}{2} (1 + \cos \theta)$

b) For $E_z = E_0 (E_0 + E_0 e^{i\theta}) e^{i\theta}$

$$|E_z| = \frac{1}{2} (1 + \cos \theta) = \frac{1}{2} (1 + \cos \theta)$$

c) For $E_z = E_0 (E_0 + E_0 e^{i\theta}) e^{i\theta}$, $|E_z| = \frac{1}{2} (1 + \cos \theta)$

Ex-23 From Eq. (12-74) $E = \frac{1}{2} \frac{E_0}{R} \left(\frac{1}{R} \right) [E_0 (1 + E_0 e^{i\theta})] e^{i\theta}$

$$E = \frac{1}{2} \frac{E_0}{R} E = \frac{1}{2} \frac{E_0}{R} \left(\frac{1}{R} \right) [-E_0 e^{i\theta} + E_0 e^{i\theta}] e^{i\theta}$$

For circular polarization, $E = \frac{1}{2} E_0$

a) $E = \frac{1}{2} E_0 = \frac{1}{2} \frac{E_0}{R} E = \frac{1}{2} \frac{E_0}{R} \left(\frac{1}{R} \right) [-E_0 e^{i\theta} + E_0 e^{i\theta}] e^{i\theta}$

$$= \frac{1}{2} \frac{E_0}{R} e^{i\theta}$$

$$E = \frac{1}{2} \frac{E_0}{R} e^{i\theta} = \frac{1}{2} \frac{E_0}{R} e^{i\theta}$$

$$\therefore E_0 = \frac{1}{2} \frac{E_0}{R} = \frac{1}{2} \frac{E_0}{R} \implies R = \frac{1}{2} \frac{E_0}{R} = \frac{1}{2} \frac{E_0}{R}$$

b) $E_0 = \frac{1}{2} \frac{E_0}{R} = \frac{1}{2} \frac{E_0}{R} \implies R = \frac{1}{2} \frac{E_0}{R} = \frac{1}{2} \frac{E_0}{R}$

Sol-13 Assume $E_0(x, y) = E_0 y$.

$$\begin{aligned} E(x, y) &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} E_0(x, y) e^{j(k_x x + k_y y)} dx dy \\ &= E_0 \left[\frac{\sin \left(\frac{k_x a}{2} \right)}{\left(\frac{k_x}{2} \right)} \right] \left[\frac{\sin \left(\frac{k_y b}{2} \right)}{\left(\frac{k_y}{2} \right)} \right] \\ E_0(x, y) &= \frac{1}{ab} e^{j\pi x} \left[\frac{\sin \left(\frac{k_x a}{2} \right)}{\left(\frac{k_x}{2} \right)} \right] \left[\frac{\sin \left(\frac{k_y b}{2} \right)}{\left(\frac{k_y}{2} \right)} \right] e^{j\pi y} = E_0(x, y) \end{aligned}$$

Sol-14 From Eq. (1) we have

$$E(x, y) = \iint_{-\infty}^{\infty} E_0(x, y) e^{j(k_x x + k_y y)} dx dy$$

a) In the xy -plane, $\phi = 0^\circ$

$$\begin{aligned} E_{xy}(x) &= b \int_{-\infty}^{\infty} f(x) e^{jk_y y} dy \\ &= b \int_{-\infty}^{\infty} \left(\frac{1}{2} e^{j\pi y} \right) \sin \left(\frac{k_y b}{2} \right) dy \\ &= ab \left[\frac{\sin \left(\frac{k_y b}{2} \right)}{\left(\frac{k_y}{2} \right)} \right] e^{j\pi y} \quad \text{Let } \phi = \frac{k_y b}{2} = \frac{\pi}{2} \text{ then } \frac{k_y}{2} = \pi \\ E_{xy}(x) &= \frac{ab}{2} \left[\frac{\sin \left(\frac{k_y b}{2} \right)}{\left(\frac{k_y}{2} \right)} \right] \end{aligned}$$

b) Let $\left[\frac{\sin \left(\frac{k_y b}{2} \right)}{\left(\frac{k_y}{2} \right)} \right] = \frac{1}{2}$ then $\frac{k_y}{2} = 2 \text{ rad}$.

Half-power beamwidth $(\Delta \alpha)_{HP} = 2 \sin^{-1} \left(\frac{k_y}{2} \right)$

$$\begin{aligned} \Delta \alpha_{HP} &= 2 \sin^{-1} \left(\frac{k_y}{2} \right) = 2 \sin^{-1} \left(\frac{1}{2} \right) \text{ rad} \\ &= 70.5^\circ \text{ (broad angle)} \end{aligned}$$

c) Let $\frac{k_y}{2} = \pi$ then $k_y = \sin^{-1} \left(\frac{k_y}{2} \right) = \frac{2\pi}{b} \text{ rad}$
 $= 2\pi \sin^{-1} \left(\frac{k_y}{2} \right)$

d) First sidelobe occurs at $\frac{k_y}{2} = 3 \text{ rad}$,

$$\text{where } E_{sl} = \frac{ab}{2} \left[\frac{1}{3} \right]^2$$

$\therefore$ Level of first sidelobe, $E_{sl} = \text{ratio} \left[\frac{1}{9} \right] = 11.25 \text{ dB}$

Comparison of Results

	Uniform State	Triangular State
Surface Recursion	$\ln\left(\frac{2\sqrt{3}E}{\pi}\right)$	$\frac{2\sqrt{3}}{\pi}\left(\frac{2\pi E}{3}\right)$
Half-power bandwidth	$2\pi\frac{1}{\sqrt{3}}\text{ rad}$	$2\pi\left(1-\frac{1}{\sqrt{3}}\right)\text{ rad}$
Location of first null	$2\pi\left(1+\frac{1}{\sqrt{3}}\right)\text{ rad}$	$2\pi\left(1+\frac{1}{\sqrt{3}}\right)\text{ rad}$
First-null side lobe level	-11.2 dB	-24.7 dB

EX-11 a) In the rectangle, $\phi = 0^\circ$:

$$\begin{aligned}
 E_{\text{avg}} &= 1.1 \int_0^{2\pi} \cos\left(\frac{\phi}{\sqrt{3}}\right) \cos(\phi) d\phi \\
 &= \frac{2\sqrt{3}}{\pi} \left[\frac{\sin\left(\frac{\phi}{\sqrt{3}}\right) \cos\phi}{\frac{1}{\sqrt{3}}} + \frac{\sin\phi}{1} \right] \quad \phi = \frac{\pi}{2} \text{ rad} = \frac{90^\circ}{1} \text{ rad}
 \end{aligned}$$

$$\text{b) Set } \frac{\left(\frac{E}{\sqrt{3}}\right) \cos\phi}{\left(\frac{E}{\sqrt{3}}\right) - \phi^2} = \frac{1}{\sqrt{3}} \quad \phi = 1.41 \text{ rad}$$

$$\text{Half-power bandwidth } (2\omega)_{\text{HP}} = 2 \sin^{-1} \left(\sin\left(\frac{\phi}{\sqrt{3}}\right) \right)$$

$$\begin{aligned}
 \text{For } \phi = 1.41 \text{ rad, } (2\omega)_{\text{HP}} &= 2 \sin^{-1} \left(\frac{1}{\sqrt{3}} \right) \text{ (rad)} \\
 &= 2.2 \left(\frac{\pi}{180} \right) \text{ (deg)}
 \end{aligned}$$

$$\begin{aligned}
 \text{c) Set } \phi &= \frac{\pi}{2} \quad \omega_{\text{null}} = \sin^{-1} \left(\frac{\frac{\pi}{2}}{\sqrt{3}} \right) = \frac{\pi}{2\sqrt{3}} \text{ (rad)} \\
 &= 16.7 \left(\frac{\pi}{180} \right) \text{ (deg)}
 \end{aligned}$$

d) At first nulls, $\phi = \pi$:

$$L_{\text{null}} = -20 \log_{10} \frac{\left(\frac{E}{\sqrt{3}}\right)}{\left(\frac{E}{\sqrt{3}}\right) - \pi^2} = 20 \log_{10} \pi = 22.8 \text{ dB}$$

	Uniform State	Triangular State
Surface Recursion	$\ln\left(\frac{2\sqrt{3}E}{\pi}\right)$	$\frac{2\sqrt{3}}{\pi}\left(\frac{2\pi E}{3}\right)$
Half-power bandwidth	$2\pi\frac{1}{\sqrt{3}}\text{ rad}$	$2\pi\left(1-\frac{1}{\sqrt{3}}\right)\text{ rad}$
Location of first null	$2\pi\left(1+\frac{1}{\sqrt{3}}\right)\text{ rad}$	$2\pi\left(1+\frac{1}{\sqrt{3}}\right)\text{ rad}$
First-null side lobe level	-11.2 dB	-24.7 dB

P. 281, Eq. (20-22): $\vec{E} \rightarrow \vec{E}'$ (2 altered); P. 281-282: $\vec{E} \rightarrow \vec{E}'$

P. 281, problem 2.22-44: last line Change "the electric" to "a";
and last before "made of copper".

P. 281, problem 2.22-44(a): Change "result" to "maximum".

P. 281, paragraph 3, line 1: $\left(\frac{E}{\epsilon_0}\right) \rightarrow \left(\frac{E}{\epsilon_0}\right)$

P. 281, last, notes, line 1: 2.22.3 $\rightarrow$ 2.22.5.

P. 281, line 10: $\frac{1}{2} q \rightarrow \frac{1}{2} q_0$

P. 281, problem 2.22-45: $\vec{E}_{\text{net}}$

(more space)

P. 281, problem 2.22-45: $\vec{E}_{\text{net}}$ and $\vec{E}_{\text{net}}$.

P. 281, problem 2.22-45: $\vec{E}_{\text{net}}$ and $\vec{E}_{\text{net}}$.

both unknown: $\vec{E}_{\text{net}}$ and $\vec{E}_{\text{net}}$ (use Eq. 22.1, p. 22 and Eq. 22.2, p. 22.)
Complete solution.

Suggest to check the explanation of what constants are used

2.2, Fig. 1-1: Polarized line should be centered on the circle.

2.2, Fig. 1-1:

2.2, Fig. 1-1(a): $\vec{E}$ should be on the circle.

2.2, Fig. 1-1(a):

2.2, Fig. 1-1: $\vec{E}$ should be on the circle.

2.2, Fig. 1-1: $\vec{E}$ should be on the circle; the arrow should be
drawn up to touch $\vec{E}$; the point on $\vec{E}$ should be $\vec{E}_{\text{net}}$ (see Fig. 2-14 on p. 22).

2.2, Fig. 1-1 (the vector) $\vec{E}_{\text{net}}$ should coincide with the $\vec{E}_{\text{net}}$
value. (see Fig. 2-14 on p. 22).

2.2, Fig. 1-1: The arrow should pass through the center of the circle.

2.2, Fig. 1-1: 2.2, Fig. 1-1: 2.2, Fig. 1-1: 2.2, Fig. 1-1.